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PRE-LINDELÖF QUASI-PSEUDO-METRIC AND QUASI-UNIFORM SPACES

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Abstract. When the finiteness condition in the definition of precompactness is replaced by countability, a new notion in the class of quasi-uniform spaces is introduced, and we call it *pre-Lindelöfness*. Some properties of pre-Lindelöf spaces, in particular in the subclass of quasi-pseudo-metric spaces, are investigated.

1. Introduction

Recently, extensive studies of precompact quasi-uniform spaces have been undertaken. Precompactness is a weaker form of generalised total boundedness. Let us recall that a quasi-uniform space $(X,\mathcal{U})$ is called *totally bounded* if for each $U \in \mathcal{U}$ there is a finite cover $\{\, A_i \,:\, i = 1,\dots,n \,\}$ of X such that $A_i \times A_i \subset U$ for each $i = 1,\dots,n$. Furthermore $(X,\mathcal{U})$ is called *precompact* if for each $U \in \mathcal{U}$ there is a finite subset $F \subset X$ such that $X = \bigcup_{x\in F} U(x) = U(F)$. As usual $U(x) = \{\, y \in X \,:\, (x,y) \in U \,\}$.

If the finiteness condition in the definition of precompactness is replaced by countability, a new property of quasi-uniform spaces is introduced, and we call it *pre-Lindelöfness.*

Definition 1. Let $(X,\mathcal{U})$ be a quasi-uniform space. Then $(X,\mathcal{U})$ is called *pre-Lindelöf* if for each $U \in \mathcal{U}$ there is a countable subset $C \subset X$ such that $X = \bigcup_{c\in C} U(c) = U(C)$. A space X is *hereditarily pre-Lindelöf* if each subspace of X is pre-Lindelöf.

In the class of uniform spaces the pre-Lindelöf property is called by different names, one of which is *trans-separability.* (See for example [8].) This property is strictly weaker than both Lindelöfness and separability, and has proved useful in investigations of metrisability of topological vector spaces.

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All (hereditarily) Lindelöf spaces and all (hereditarily) precompact spaces are (hereditarily) pre-Lindelöf, but the converse is not true in general, except for hereditary Lindelöfness in the class of quasi-pseudo-metric spaces as Theorem 2 shows.

As noted in [7], the usual metric on the set of real numbers gives a space which is hereditarily Lindelöf but not precompact, while the Sorgenfrey plane is pre-Lindelöf but not Lindelöf. The Sorgenfrey line quasi-metric restricted to the interval $[0, 1]$ is precompact. Its square is precompact but not Lindelöf.

In the class of quasi-pseudo-metric spaces pre-Lindelöfness has some nice properties. We prove (Theorem 1) that pre-Lindelöfness is equivalent to separability of the conjugate space. Also, we show that hereditary pre-Lindelöfness is equivalent to the hereditary Lindelöf property (Theorem 2). Neither of these two statements is true in general in the class of quasi-uniform spaces.

Our notation is standard. In particular, $\mathbf{R}$ and $\mathbf{N}$ denote the sets of real numbers and positive integers, respectively. In a quasi-pseudo-metric space (X, d) the ball with centre x and radius ε is the set $\{\, y \in X \mid d(x, y) < \varepsilon \,\}$ and is denoted by $B_d(x, \varepsilon)$, although we suppress the 'd' when there is no possible confusion. If d is a quasi-pseudo-metric on X, then so is d' defined by $d'(x, y) = d(y, x)$ for $x, y \in X$, and d' is called the *conjugate* of d.

2. Pre-Lindelöf quasi-pseudo-metric spaces.

In the subclass of quasi-pseudo-metric spaces pre-Lindelöfness can be defined in the following way.

DEFINITION 2. A quasi-pseudo-metric space (X, d) is *pre-Lindelöf* if for every $\varepsilon > 0$ there is a countable subset $C \subset X$ such that $X \subset \bigcup_{c \in C} B(c, \varepsilon)$.

The duality between pre-Lindelöfness and separability is given by the following result.

THEOREM 1. *A quasi-pseudo-metric space (X, d) is pre-Lindelöf if and only if its conjugate space (X, d') is separable.*

Proof. Let (X, d) be pre-Lindelöf. For each $n \in \mathbf{N}$ there is a countable subset $A_n \subset X$ such that $X = \bigcup_{a \in A_n} B_d(a, \frac{1}{n})$. The set $A = \bigcup_{n \in \mathbf{N}} A_n$ is countable and dense in (X, d').

Conversely, let D be a countable dense subset in (X, d'). Let $\varepsilon > 0$ and let $x \in X$. There is an $a \in D$ such that $d'(x, a) < \varepsilon$, which implies $d(a, x) < \varepsilon$. Thus $X = \bigcup_{a \in D} B_d(a, \varepsilon)$, so that (X, d) is a pre-Lindelöf space. ■

COROLLARY 1. *In a pseudometric space the following are equivalent*:

(i) *second countability*,

(ii) *separability*,

(iii) *Lindelöfness*,

(iv) *ccc (the countable chain condition)*,

(v) *pre-Lindelöfness.* ■

COROLLARY 2. *If Y is an open subset in the conjugate topology of a quasi-pseudo-metric space (X, d_X), then (X, d_X) is pre-Lindelöf implies (Y, d_Y) is pre-Lindelöf.* ■

While pre-Lindelöfness is in general strictly weaker than the Lindelöf property, their hereditary counterparts coincide in the class of quasi-pseudo-metric spaces, as the following result shows.

THEOREM 2. *A quasi-pseudo-metric space is hereditarily pre-Lindelöf if and only if it is hereditarily Lindelöf.*

Proof. Only the "only if" part has to be proved. For this we modify the proof of Lemma 2 in [7].

Let $E \subset X$ and let $\mathcal{G}$ be a collection of open sets in X such that $E \subset \bigcup\{\, G \mid G \in \mathcal{G}\,\}$. For each $x \in E$ choose $G(x) \in \mathcal{G}$ and $n(x) \in \mathbf{N}$ such that $B(x, 1/n(x)) \subset G(x)$. For each $n \in \mathbf{N}$ let $A_n = \{\, x \in E \mid n(x) = n\,\}$. For each $n \in \mathbf{N}$ there is countable subset C_n of A_n such that $A_n \subset \bigcup_{c \in C_n} B(c, \frac{1}{n})$. Then $E \subset \bigcup\{\, B(c, \frac{1}{n}) \mid c \in C_n$ and $n \in \mathbf{N}\,\}$. It follows that E is a Lindelöf subset of X. ■

Since both hereditary Lindelöfness and separability imply the countable chain condition, we have the following

COROLLARY 3. *Let (X, d) be a hereditarily (pre-)Lindelöf quasi-pseudo-metric space. Then both (X, d) and (X, d') are hereditarily ccc.* ■

Note that Example 5 in [7] gives a space (X, s) which is hereditarily pre-compact, thus hereditarily (pre-)Lindelöf, but not separable. Thus hereditary (pre-)Lindelöfness and separability are not comparable; also, neither separability nor hereditary (pre-)Lindelöfness is equivalent to the ccc in quasi-pseudo-metric spaces.

COROLLARY 4. [4, Theorem 4] *A quasi-pseudo-metric space is hereditarily (pre-)Lindelöf if and only if its conjugate space is hereditarily separable.* ■

We are able to define a more general version of the pre-Lindelöf property given in Definition 2.

DEFINITION 3. Let (X, d) be a quasi-pseudo-metric space. Given $D \subset X$ and $\varepsilon > 0$, we define (X, d) to be (D, ε)-*Lindelöf* if $X = \bigcup_{x \in D} B_d(x, \varepsilon)$.

DEFINITION 4. Let (X, d) be a quasi-pseudo-metric space. Given $D \subset X$ and $\varepsilon > 0$, we define D to be ε-*dense* in (X, d), if for every $x \in X$ there is a point $y \in D \cap B_d(x, \varepsilon)$.

REMARK. (X, d) is pre-Lindelöf (pre-compact) if for every $\varepsilon > 0$ there is a countable (finite) subset D such that (X, d) is (D, ε)-Lindelöf.

THEOREM 3. *(X, d) is separable if and only if for every $\varepsilon > 0$ there is a countable subset D which is ε-dense in (X, d).*

Proof. The "only if" part is obvious. For the converse, for every $n \in \mathbf{N}$ there is a countable subset D_n such that D_n is $\frac{1}{n}$-dense in (X, d). Let $D = \bigcup_{n \in \mathbf{N}} D_n$. Then

D is countable. We claim that D is dense in (X,d). Let $x \in X$ and $\varepsilon > 0$ be given. Choose $n \in \mathbf{N}$, with $\frac{1}{n} < \varepsilon$. Then there is a point $y \in D_n \cap B_d(x, \frac{1}{n}) \subset D \cap B_d(x,\varepsilon)$. ∎

Theorem 4. *Let d, d' be conjugate quasi-pseudo-metrics on X and let $D \subset X$, $\varepsilon > 0$. Then (X,d) is (D,ε)-Lindelöf if and only if D is ε-dense in (X,d').*

Proof. Let $x \in X$, $y \in D$. Then $x \in B_d(y,\varepsilon)$ if and only if $y \in B_{d'}(x,\varepsilon)$. ∎

3. Pre-Lindelöf quasi-uniform spaces.

When the class of quasi-pseudo-metric spaces is enlarged to the class of quasi-uniform spaces, pre-Lindelöfness behaves in a slightly different way. Theorems 1 and 2 no longer hold in general as the following examples show.

Example 1. Let X be an uncountable set endowed with the cocountable topology $\mathcal{T}$. Let $\mathcal{U}$ be the Pervin quasi-uniformity compatible with $\mathcal{T}$. The space $(X,\mathcal{U})$ is totally bounded, thus both $(X,\mathcal{U})$ and its conjugate $(X,\mathcal{U}^{-1})$ are hereditarily precompact, hence hereditarily pre-Lindelöf. The topology $\mathcal{T}(\mathcal{U}^{-1})$ is discrete, thus the space $(X,\mathcal{U}^{-1})$ is neither separable nor Lindelöf. Furthermore $(X,\mathcal{T}) = (X,\mathcal{T}(\mathcal{U}))$ is not separable. Note that $(X,\mathcal{U})$ has the ccc while $(X,\mathcal{U}^{-1})$ has not.

Example 2. For any topological space X the semi-continuous quasi-uniformity $\mathcal{S}(X)$ has the property that $\mathcal{S}(X)^* = \sup\{\mathcal{S}(X), \mathcal{S}(X)^{-1}\}$ is (hereditarily) pre-Lindelöf. (See [2, Theorem 2.12] and Proposition 2.) Thus Theorem 2 does not hold even in the class of uniform spaces.

Precompactness is a productive property as stated in [2] for quasi-uniform spaces. Theorem 5 shows that pre-Lindelöfness is also productive. In Theorem 5 of [7] it was proved that hereditary precompactness is productive as well, while the Sorgenfrey plane shows a different behaviour of hereditary pre-Lindelöfness. By Theorem 2, the Sorgenfrey plane being quasi-metrisable is not hereditarily pre-Lindelöf, thus hereditary pre-Lindelöfness is not a productive property. Theorem 6 gives a necessary and sufficient condition for a product of infinitely many quasi-uniform factor spaces to be hereditarily pre-Lindelöf.

The next two results are given without proofs which are straightforward.

Proposition 1. *Let $(X,\mathcal{U})$ and $(Y,\mathcal{V})$ be quasi-uniform spaces and let $f\colon (X,\mathcal{U}) \to (Y,\mathcal{V})$ be a quasi-uniformly continuous function. If $(X,\mathcal{U})$ is (hereditarily) pre-Lindelöf, then so is $(f(X), \mathcal{V}_{f(X)})$.* ∎

Proposition 2. *Pre-Lindelöfness is hereditary in uniform spaces.* ∎

Theorem 5. *The nonempty product of a family of quasi-uniform spaces is pre-Lindelöf if and only if each factor space is pre-Lindelöf.*

Proof. The "only if" part follows from Proposition 1. For the converse, let $(X,\mathcal{U})$ be the product of a family $\{(X_i,\mathcal{U}_i)\}_{i\in I}$ of pre-Lindelöf quasi-uniform spaces. For each $j \in I$ let $\mathrm{pr}_j : \prod_{i\in I} X_i \to X_j$ denote the projection. Suppose that $U = \bigcap_{k=1}^{n} (\mathrm{pr}_{i_k} \times \mathrm{pr}_{i_k})^{-1}(V_{i_k})$, where $V_{i_k} \in \mathcal{U}_{i_k}$, $k = 1,\dots,n$, and $n \in \mathbf{N}$. For each

V_{i_k} there is a countable subset $C_{i_k} \subset X_{i_k}$ such that $X_{i_k} = \bigcup_{c \in C_{i_k}} V_{i_k}(c)$. For each $i \in I \setminus \{i_1, \ldots, i_n\}$ choose a point $a_i \in X_i$. Let C be a subset of X consisting of all points $c \in X$ such that $c_i \in C_i$ if $i \in \{i_1, \ldots, i_n\}$, and $c_i = a_i$ otherwise. The subset C is countable and $X = U(C)$. Therefore $\prod_{i \in I} X_i$ is pre-Lindelöf. ■

THEOREM 6. *The nonempty product of a family* $\{(X_i, \mathcal{U}_i)\}_{i \in I}$ *of hereditarily pre-Lindelöf quasi-uniform spaces is hereditarily pre-Lindelöf if and only if the product* $\prod_{i \in F} X_i$ *is hereditarily pre-Lindelöf for any nonempty finite subset* F *of* I.

Proof. Since hereditary pre-Lindelöfness is preserved by quasi-uniformly continuous surjections, the "only if" part is clear. For the converse, let A be a subspace of $\prod_{i \in I} X_i$. For each $j \in I$ let $\mathrm{pr}_j \colon \prod_{i \in I} X_i \to X_j$ denote the projection. Suppose that $U = \bigcap_{k=1}^{n} (\mathrm{pr}_{i_k} \times \mathrm{pr}_{i_k})^{-1}(V_{i_k})$, where $V_{i_k} \in \mathcal{U}_{i_k}$, $k = 1, \ldots, n$, and $n \in \mathbf{N}$. Let $p : \prod_{i \in I} X_i \to \prod_{k=1}^{n} X_{i_k}$ be the projection. For simplicity the projections of the latter product onto its factor spaces are also denoted by pr_i. Since $\prod_{k=1}^{n} X_{i_k}$ is hereditarily pre-Lindelöf, there is a countable subset $C \subset p(A)$ such that $p(A) \subset \bigcup_{c \in C} [V_{i_1}(\mathrm{pr}_{i_1}(c)) \times \cdots \times V_{i_n}(\mathrm{pr}_{i_n}(c))]$. For each $c \in C$ choose $d_c \in A$ such that $p(d_c) = c$. Therefore $A \subset \bigcup_{c \in C} U(d_c)$ and we conclude that $\prod_{i \in I} X_i$ is hereditarily pre-Lindelöf. ■

PROPOSITION 3. *Let* $(X, \mathcal{U})$ *be a quasi-uniform space such that* $\sup\{\mathcal{U}, \mathcal{U}^{-1}\}$ *is (hereditarily) pre-Lindelöf and let* $(Y, \mathcal{V})$ *be a hereditarily pre-Lindelöf quasi-uniform space. Then* $(X \times Y,\ \mathcal{U} \times \mathcal{V})$ *is hereditarily pre-Lindelöf.*

Proof. Let $B \subset X \times Y$ and let $U \in \mathcal{U}$ and $V \in \mathcal{V}$. Since $\sup\{\mathcal{U}, \mathcal{U}^{-1}\}$ is pre-Lindelöf, there exists a countable cover $(A_i)_{i \in \mathbf{N}}$ of $\{\, x \in X \mid (x, y) \in B \,\}$ such that $A_i \times A_i \subset U$ whenever $i \in \mathbf{N}$. For each $i \in \mathbf{N}$ set $D_i = \{\, y \in Y \mid \text{there is } x \in A_i \text{ with } (x, y) \in B \,\}$. Since $(Y, \mathcal{V})$ is hereditarily pre-Lindelöf, for each $i \in \mathbf{N}$ there is a countable set $C_i \subset D_i$ such that $D_i \subset V(C_i)$. For each $i \in \mathbf{N}$ and each $c \in C_i$ choose $x_c \in A_i$ such that $(x_c, c) \in B$. Set $C = \{\, (x_c, c) \mid c \in C_i, i \in \mathbf{N} \,\}$. Let $(x, y) \in B$. There is $i_0 \in \mathbf{N}$ such that $x \in A_{i_0}$. Therefore $y \in D_{i_0}$ and $y \in V(c)$ for some $c \in C_{i_0}$. Thus $(x, y) \in A_{i_0} \times V(c) \subset U(x_c) \times V(c)$. We conclude that $(X \times Y,\ \mathcal{U} \times \mathcal{V})$ is hereditarily pre-Lindelöf. ■

THEOREM 7. *The product* $(X \times Y,\ \mathcal{U} \times \mathcal{V})$ *of a hereditarily pre-Lindelöf quasi-uniform space* $(X, \mathcal{U})$ *and a hereditarily precompact quasi-uniform space* $(Y, \mathcal{V})$ *is hereditarily pre-Lindelöf.*

Proof. Suppose that a subspace B of $X \times Y$ is not pre-Lindelöf. Then there are $U \in \mathcal{U}$ and $V \in \mathcal{V}$ such that $B \subset \bigcup_{a \in A} W(a)$ and $A \subset B$ imply that A is uncountable, where $W = [(\mathrm{pr}_X \times \mathrm{pr}_X)^{-1}(U)] \cap [(\mathrm{pr}_Y \times \mathrm{pr}_Y)^{-1}(V)]$ and $\mathrm{pr}_X \colon X \times Y \to X$ and $\mathrm{pr}_Y \colon X \times Y \to Y$ denote the projections. Inductively construct $C = \{\, x_\alpha \mid \alpha < \omega_1 \,\}$ such that $x_\alpha \in B \setminus W(\{\, x_\beta \mid \beta < \alpha \,\})$ whenever $\alpha < \omega_1$. Define a graph with vertex set C and edges $\{x_\alpha, x_\beta\}$ provided that $\alpha < \beta < \omega_1$ and $\mathrm{pr}_X(x_\beta) \in U(\mathrm{pr}_X(x_\alpha))$. Since $(X, \mathcal{U})$ is hereditarily pre-Lindelöf, for each uncountable $D \subset C$ there are two vertices of D connected by an edge. By [1, Theorem 5.22], there is a countably infinite subset E of C such that any two vertices of E are connected. Hence if $\alpha < \beta < \omega_1$ and $x_\alpha, x_\beta \in E$, then $\mathrm{pr}_Y(x_\beta) \notin V(\mathrm{pr}_Y(x_\alpha))$. We conclude that the first infinite initial segment of $\mathrm{pr}_Y(E)$ ordered according to the index set is not precompact, a contradiction. Therefore $X \times Y$ is hereditarily pre-Lindelöf. ∎

THEOREM 8. *A topological space X is hereditarily Lindelöf if and only if each of its compatible quasi-uniformities is hereditarily pre-Lindelöf.*

Proof. Only the "if" part needs a proof. If X is not hereditarily Lindelöf, then there is a strictly increasing sequence $(G_\alpha)_{\alpha<\omega_1}$ of open sets in X . The binary relation T defined by $T(x) = \bigcap\{ G_\alpha \mid x \in G_\alpha, \alpha < \omega_1 \}$ for all $x \in X$, where we use the convention that the intersection over the empty collection is X, belongs to the fine quasi-uniformity $\mathcal{V}$ of X. Since $\mathcal{V}$ restricted to the subspace $\bigcup_{\alpha<\omega_1} G_\alpha$ is not pre-Lindelöf, $\mathcal{V}$ is not hereditarily pre-Lindelöf. ∎

PROBLEM. Characterize those topological spaces that admit only pre-Lindelöf quasi-uniformities.

PROPOSITION 4. *If $(X,\mathcal{U})$ and $(Y,\mathcal{V})$ are hereditarily pre-Lindelöf, then each subspace A of $X \times Y$ has the property that for each $W \in \mathcal{U} \times \mathcal{V}$ there is $M \subset A$ such that $|M| \le 2^{\aleph_0}$ and $A \subset W(M)$.*

Proof. Otherwise there are $U \in \mathcal{U}$, $V \in \mathcal{V}$ and $C = \{ x_\alpha \mid \alpha < (2^{\aleph_0})^+ \}$ such that $x_\alpha \notin U(\mathrm{pr}_X(x_\beta)) \times V(\mathrm{pr}_Y(x_\beta))$ whenever $\beta < \alpha < (2^{\aleph_0})^+$. Let $\beta < \alpha < (2^{\aleph_0})^+$. If $\mathrm{pr}_X(x_\alpha) \notin U(\mathrm{pr}_X(x_\beta))$, set $f(\{x_\beta, x_\alpha\}) = 1$, and set $f(\{x_\beta, x_\alpha\}) = 0$ otherwise. Since X and Y are hereditarily pre-Lindelöf we see similarly as above that there is no uncountable subset A of C such that f is constant on $[A]^2$, contradicting the well known Erdös-Rado Partition Theorem [3, Theorem 69]. ∎

Recall that a topological space is *almost realcompact* if each maximal open filter $\mathcal{F}$ such that $\overline{\mathcal{F}} = \{ \overline{F} \mid F \in \mathcal{F} \}$ has the countable intersection property has a cluster point. The space is called *closed complete* if each maximal closed filter with the countable intersection property has a non-empty intersection. (See [6, p. 360].) A quasi-uniform space $(X,\mathcal{U})$ is called *complete* if each $\mathcal{U}$-Cauchy filter has a cluster point. (See [2, Definition 3.8].)

PROPOSITION 5. *Each pre-Lindelöf complete quasi-uniform space $(X,\mathcal{U})$ is:* (i) *almost realcompact,* (ii) *closed complete.*

Proof. (i) Let $\mathcal{F}$ be a maximal open filter in $(X,\mathcal{U})$ such that $\overline{\mathcal{F}}$ has the countable intersection property. Consider $U \in \mathcal{U}$. There is a $V \in \mathcal{U}$ such that $V(x)$ is open in $(X, \mathcal{T}(\mathcal{U}))$ whenever $x \in X$ [2, p. 3] and such that $V \subset U$. Since $\mathcal{U}$ is pre-Lindelöf, there is a countable subset C of X such that $\bigcup_{x\in C} V(x) = X$. Since $\overline{\mathcal{F}}$ has the countable intersection property, there is an $x \in C$ such that $V(x) \cap F \neq \emptyset$ whenever $F \in \mathcal{F}$. By maximality of $\mathcal{F}$, $V(x) \in \mathcal{F}$. We conclude that $\mathcal{F}$ is a $\mathcal{U}$-Cauchy filter base. Hence $\bigcap \overline{\mathcal{F}} \neq \emptyset$ because $(X,\mathcal{U})$ is complete. We have shown that $(X, \mathcal{T}(\mathcal{U}))$ is almost realcompact.

(ii) Closed completeness is proved in a similar way. ∎

An analogue of the well known result in the class of topological spaces that compactness is equivalent to countable compactness and Lindelöfness, is the following, whose straightforward proof is omitted. The statement for quasi-pseudo-metric spaces then follows from Corollary 2 of [5].

PROPOSITION 6. *A countably compact pre-Lindelöf quasi-uniform space is precompact. In particular, a countably compact pre-Lindelöf quasi-pseudo-metric space is compact.* ∎

That the converse of Proposition 6 does not hold can be seen from Example 3. Example 4 shows that the statement is not true for hereditary counterparts even in quasi-metric spaces.

EXAMPLE 3. The subspace $(0, 1)$ of the real line is precompact but not countably compact.

EXAMPLE 4. Let $X = \mathbf{N}$ and for $m, n \in X$ let

$$d(m,n) = \begin{cases} \dfrac{1}{n}, & \text{if } m = 1 \text{ and } n > 1, \\ 0, & \text{if } m = n\ , \\ 1, & \text{otherwise}\ . \end{cases}$$

The space (X, d) is compact but not hereditarily precompact.

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