MATEMATIČKI VESNIK
МАТЕМАТИЧКИ ВЕСНИК
73, 2 (2021), 75–87
June 2021

research paper
оригинални научни рад

SOFT PROXIMITY BASES AND SUBBASES

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Abstract. The purpose of this paper is to introduce the concept of soft proximity bases and subbases. We determine the relation between proximity bases (subbases) and soft proximity bases (subbases). Further, we have demonstrated that the set of all soft proximities forms a complete lattice. Also, we substantiate a few results analogous to the ones that hold for soft proximity spaces.

1. Introduction

D. Molodstov [19] initiated a different approach for working with uncertainties by introducing the novel concept of a soft set. The soft set can be used for modelling problems in computer science, engineering physics, medical science etc. This theory has several applications in many different fields and play a vital role in decision-making problems.

Maji et al. [17, 18] worked on this theory and gave the first practical application in decision-making problems. Chen et al. [3], Kong et al. [14] and then Ma et al. [16] gave their approaches for reduction of problems in soft sets. Pei and Miao [23] also contributed in the development of soft set theory. Ali et al. [1] introduced some new operations on soft sets. Soft topological spaces were introduced by Shabir and Naz [24]. Further, the research in soft topological space has been done by many mathematicians.

In 1951, Efremovic [7, 8] gave axioms for proximity and showed that a metric space and a topological group can be generalized by using proximity in a natural manner. In [8], closure of a set in terms of proximity to introduce a topology in a proximity space was defined. Many significant topological problems can be simplified by using a simple conceptual approach of proximity. For example: compactifications, the problem of continuous extensions of functions etc. Smirnov [26, 27], Naimpally and Warrack [20] did the most substantial and extensive work on proximity spaces.

2020 Mathematics Subject Classification: 54A40, 06D72, 54E05.
Keywords and phrases: Soft set; soft proximity; soft neighbourhood; soft p-continuous mapping; soft proximity bases; soft proximity subbases; induced soft proximity.

Bases for Efremovic proximity were studied by Császár et al. [4], Njastad [22], and Sharma [25]. Initially, the concept of proximity was merged with fuzzy set and later in subsequent years, with the soft set by many researchers. For the first time, Hazra et al. [9] brought out the concept of soft proximity. They [10] also defined a different concept of proximity similar to the axioms of basic proximity and termed it as proximity of soft sets. Then, Kandil et al. [11] made a base of the axioms given by Efremovic to define soft proximity spaces. After that, Kandil et al. [12] made use of an ideal to define soft I-proximity. Further, Demir et al. [6] gave his contribution to the study of soft proximity spaces in Kandil et al.'s sense [11].

In this paper, we have utilized proximity base [25] and soft set [19] to introduce soft proximity base and subbase. Multiple theorems in soft proximity spaces are remarkably simplified by applying soft proximity bases (proximity subbases). We also propose an algorithm to generate a soft proximity from soft proximity bases and subbases. Further, we identify the relation between two different soft proximities generated and induced through two different ways. Finally, we establish a few results analogous to the ones that hold for soft proximity spaces. For example, soft p-continuity and product soft proximity have been characterized in terms of soft proximity bases and subbases.

2. Preliminaries

In this section, we recap some basic results including soft set, soft topology, soft proximity space, proximity base, proximity subbase etc. Let X be an initial universe, $\mathcal{P}(X)$ be the power set of X and E be a set of parameter for X throughout this paper.

Definition 2.1. [19] A soft set F on the universe X with the set of parameters E is defined by the set of ordered pairs: $F = \{(e, F(e)) : e \in E,\ F(e) \in \mathcal{P}(X)\}$, where F is a mapping given by $F : E \to \mathcal{P}(X)$.

In this paper, the family of all soft sets over X is denoted by $S(X, E)$ [2]. It is assumed throughout this paper that a soft set means a set together with a common set of parameter E unless or otherwise stated about the set of parameter.

Definition 2.2 ([1, 17, 23]). Let $F, G \in S(X, E)$. Then:

(i) The soft set F is called null soft set, denoted by Φ, if $F(e) = \phi$ for every e in E.

(ii) If $F(e) = X$ for all e in E, then F is called absolute soft set, denoted by $\tilde{X}$.

(iii) F is a soft subset of G if $F(e) \subseteq G(e)$ for every $e \in E$. It is denoted by $F \sqsubseteq G$.

(iv) F and G are equal if $F \sqsubseteq G$ and $G \sqsubseteq F$. It is denoted by $F = G$.

(v) The complement of F is denoted by F^c, where $F^c : E \to \mathcal{P}(X)$ is a mapping defined by $F^c(e) = X \setminus F(e)$ for all e in E. Clearly, $(F^c)^c = F$.

(vi) The union of F and G is a soft set H defined by $H(e) = F(e) \cup G(e)$ for all e in E. H is denoted by $F \sqcup G$.

(vii) The intersection of F and G is a soft set H defined by $H(e) = F(e) \cap G(e)$ for all e in E. H is denoted by $F \sqcap G$.

Definition 2.3 ([5, 15, 21]). A soft set P over X is said to be a soft point if there exists e in E such that $P(e) = \{x\}$ for some x in X and $P(e') = \phi$ for all e' in $E \setminus \{e\}$. The soft point is denoted by x^e.

The set of all soft points over X is denoted by $SP(X)$.

Definition 2.4 ([5, 21]). A soft point x^e is said to be in a soft set F if $x \in F(e)$ and is denoted by $x^e \tilde{\in} F$.

Definition 2.5 ([5]). Two soft points $x_1^{e_1}$, $x_2^{e_2}$ are said to be equal if $e_1 = e_2$ and $x_1 = x_2$. Thus, $x_1^{e_1} \neq x_2^{e_2}$ if and only if $x_1 \neq x_2$ or $e_1 \neq e_2$.

Definition 2.6 ([13]). Let $S(X,E)$ and $S(Y,K)$ be the families of all soft sets over X and Y respectively. Let $\varphi : X \to Y$ and $\psi : E \to K$ be two mappings. Then the mapping φ_ψ is called a soft mapping from X to Y, denoted by $\varphi_\psi : S(X,E) \to S(Y,K)$.
(i) Let $F \in S(X,E)$. Then $\varphi_\psi(F)$ is the soft set over Y defined for all $k \in K$ as follows:

$$\varphi_\psi(F)(k) = \begin{cases} \bigcup_{e \in \psi^{-1}(k)} \varphi(F(e)) : & \text{if } \psi^{-1}(k) \neq \phi; \\ \phi : & \text{otherwise;} \end{cases}$$

$\varphi_\psi(F)$ is called a soft image of a soft set F.

(ii) Let $G \in S(Y,K)$. Then $\varphi_\psi^{-1}(G)$ is the soft set over X defined for all $e \in E$ as follows: $\varphi_\psi^{-1}(G)(e) = \varphi^{-1}(G(\psi(e)))$. $\varphi_\psi^{-1}(G)$ is called a soft inverse image of a soft set G.

The soft mapping φ_ψ is called injective, if φ and ψ are injective. The soft mapping φ_ψ is called surjective, if φ and ψ are surjective [2, 28].

Theorem 2.7 ([13]). *Let $F_i \in S(X,E)$ and $G_i \in S(Y,K)$ for all $i \in J$ where J is an index set. Then, for a soft mapping $\varphi_\psi : S(X,E) \to S(Y,K)$, the following conditions are satisfied:*
(i) If $F_1 \sqsubseteq F_2$, then $\varphi_\psi(F_1) \sqsubseteq \varphi_\psi(F_2)$. *(ii) If $G_1 \sqsubseteq G_2$, then $\varphi_\psi^{-1}(G_1) \sqsubseteq \varphi_\psi^{-1}(G_2)$.*
(iii) $\varphi_\psi(\bigsqcup_{i \in J} F_i) = \bigsqcup_{i \in J} \varphi_\psi(F_i)$. *(iv) $\varphi_\psi^{-1}(\bigsqcup_{i \in J} G_i) = \bigsqcup_{i \in J} \varphi_\psi^{-1}(G_i)$.*
(v) $\varphi_\psi^{-1}(\bigsqcap_{i \in J} G_i) = \bigsqcap_{i \in J} \varphi_\psi^{-1}(G_i)$. *(vi) $\varphi_\psi^{-1}(\tilde{Y}) = \tilde{X}$, $\varphi_\psi^{-1}(\Phi) = \Phi$, $\varphi_\psi(\Phi) = \Phi$.*

Theorem 2.8 ([2, 28]). *Let $F, F_i \in S(X,E)$ for all $i \in J$, where J is an index set and let $G \in S(Y,K)$. Then, for a soft mapping $\varphi_\psi : S(X,E) \to S(Y,K)$, the following conditions are satisfied:*
(i) $F \sqsubseteq \varphi_\psi^{-1}(\varphi_\psi(F))$, the equality holds if φ_ψ is injective.
(ii) $\varphi_\psi(\varphi_\psi^{-1}(G)) \sqsubseteq G$, the equality holds if φ_ψ is surjective.

Definition 2.9 ([2]). Let $F \in S(X,E)$, $G \in S(Y,K)$ and let $p_X : X \times Y \to X$, $q_E : E \times K \to E$ and $p_Y : X \times Y \to Y$, $q_K : E \times K \to K$ be the projection mappings in classical meaning. The soft mapping $(p_X)_{q_E}$ and $(p_Y)_{q_K}$ are called soft projection mappings from $X \times Y$ to X and from $X \times Y$ to Y respectively, where $(p_X)_{q_E}(F \times G) = F$ and $(p_Y)_{q_K}(F \times G) = G$ respectively.

Definition 2.10 ([24]). Let $\mathcal{T}$ be a collection of soft sets over X. Then $\mathcal{T}$ is said to be a soft topology on X if:
ST(i) Φ, $\tilde{X}$ belong to $\mathcal{T}$;

ST(ii) the union of any number of soft sets in $\mathcal{T}$ belongs to $\mathcal{T}$;

ST(iii) the intersection of any two soft sets in $\mathcal{T}$ belongs to $\mathcal{T}$.

Here $(X, \mathcal{T}, E)$ is called a soft topological space. The members of $\mathcal{T}$ are called soft open sets in X. A soft set F over X is soft closed if $F^c \in \mathcal{T}$.

Definition 2.11 ([24]). Let $(X, \mathcal{T}, E)$ be soft topological space and $F \in S(X, E)$. The soft closure of F is the soft set $\overline{F} = \sqcap\{G : G \text{ is a soft closed set and } F \sqsubseteq G\}$.

Theorem 2.12. *[21] Let us consider an operator which associates each soft set F on X to another soft set $\overline{F}$ such that the following properties hold:*
SO(i) $F \sqsubseteq \overline{F}$; *SO(ii)* $\overline{\overline{F}} = \overline{F}$; *SO(iii)* $\overline{F \sqcup G} = \overline{F} \sqcup \overline{G}$; *SO(iv)* $\overline{\Phi} = \Phi$.
Then the family $\mathcal{T} = \{F \in S(X, E) : \overline{F^c} = F^c\}$ forms a soft topology on X and for every $F \in S(X, E)$, the soft set $\overline{F}$ is the soft closure of F in the soft topological space $(X, \mathcal{T}, E)$.

This operator is called soft closure operator.

Definition 2.13 ([20]). A binary relation δ on $\mathcal{P}(X)$ is called a proximity on X, if the following axioms are satisfied for all A, B, C in $\mathcal{P}(X)$:
P(i) $(\varnothing, A) \notin \delta$;

P(ii) If $A \cap B \neq \varnothing$, then $(A, B) \in \delta$;

P(iii) If $(A, B) \in \delta$, then $(B, A) \in \delta$;

P(iv) $(A, B \cup C) \in \delta$ if and only if $(A, B) \in \delta$ or $(A, C) \in \delta$;

P(v) If $(A, B) \notin \delta$, then there exists a subset C of X such that $(A, C) \notin \delta$ and $(X \setminus C, B) \notin \delta$.

Definition 2.14 ([25]). A binary relation δ_1 is said to be finer than a binary relation δ_2 if $(A, B) \in \delta_1$, then $(A, B) \in \delta_2$ for all subsets A, B of X. We write it as $\delta_1 \geq \delta_2$.

Definition 2.15 ([25]). Let X be a non-empty set. A proximity base on X is a binary relation β on $\mathcal{P}(X)$ satisfying the following axioms for all A, B, C in $\mathcal{P}(X)$:
B(i) $(\varnothing, A) \notin \beta$;

B(ii) If $A \cap B \neq \varnothing$, then $(A, B) \in \beta$;

B(iii) If $(A, B) \in \beta$, then $(B, A) \in \beta$;

B(iv) If $(A, B) \in \beta$ and $A \subseteq A^*$, $B \subseteq B^*$, then $(A^*, B^*) \in \beta$;

B(v) If $(A, B) \notin \beta$, then there exists a subset C of X such that $(A, C) \notin \beta$ and $(X \setminus C, B) \notin \beta$.

Definition 2.16 ([25]). Let X be a non-empty set. A proximity subbase on X is a binary relation s on $\mathcal{P}(X)$ satisfying the following axioms for all A, B, C in $\mathcal{P}(X)$:
S(i) If $A \cap B \neq \varnothing$, then $(A, B) \in s$;

S(ii) If $(A, B) \notin s$, then there exists a subset C of X such that $(A, C) \notin s$ and $(X \setminus C, B) \notin s$.

Definition 2.17 ([11]). A binary relation δ on $S(X, E)$ is called a proximity of soft sets on X if for any $F, G, H \in S(X, E)$, the following conditions are satisfied:
SP(i) $(\Phi, F) \notin \delta$;

SP(ii) If $F \sqcap G \neq \Phi$, then $(F, G) \in \delta$;

SP(iii) If $(F, G) \in \delta$, then $(G, F) \in \delta$;

SP(iv) $(F, G \sqcup H) \in \delta$ if and only if $(F, G) \in \delta$ or $(F, H) \in \delta$;

SP(v) If $(F, G) \notin \delta$, then there exists $H \in S(X, E)$ such that $(F, H) \notin \delta$ and $(\tilde{X} \setminus H, G) \notin \delta$.
A soft proximity space (X, δ, E) consists of a set X, a set of parameters E and a proximity relation δ on $S(X, E)$. We say two soft sets F and G are δ-related if $(F, G) \in \delta$, otherwise they are not δ-related.

Definition 2.18 ([6]). Let (X, δ, E) be a soft proximity space. For $F, G \in S(X, E)$, the soft set G is said to be a soft δ-neighbourhood of F if $(F, \tilde{X} \setminus G) \notin \delta$. We write this in symbols as $F \Subset G$.

Definition 2.19 ([6]). Let (X, δ_1, E) and (Y, δ_2, K) be two soft proximity spaces. A soft mapping $\varphi_\psi : (X, \delta_1, E) \longrightarrow (Y, \delta_2, K)$ is soft p-continuous if it satisfies: $(F, G) \in \delta_1$ implies $(\varphi_\psi(F), \varphi_\psi(G)) \in \delta_2$ for all F, G in $S(X, E)$.

3. Bases and subbases for soft proximity

In this section, first we give a definition of soft proximity base which is a generalization of the notion of proximity base [25] to the soft set.

Definition 3.1. Let X be a non-empty set. A soft proximity base on X is a binary relation β on $S(X, E)$ that satisfy the following axioms for all $F, G, H \in S(X, E)$:
SB(i) $(\Phi, F) \notin \beta$;

SB(ii) If $F \sqcap G \neq \Phi$, then $(F, G) \in \beta$;

SB(iii) If $(F, G) \in \beta$, then $(G, F) \in \beta$;

SB(iv) If $(F, G) \in \beta$ and $F \sqsubseteq F^*$, $G \sqsubseteq G^*$, then $(F^*, G^*) \in \beta$;

SB(v) If $(F, G) \notin \beta$, then there exists a soft set H in $S(X, E)$ such that $(F, H) \notin \beta$ and $(\tilde{X} \setminus H, G) \notin \beta$.

Definition 3.2. A soft proximity base β on X is separated if $(x^e, y^f) \in \beta$ implies $x = y$ and $e = f$ for all $x^e, y^f \in S(X, E)$.

Now in the next theorem, we get a relation between proximity base and the soft proximity base.

Theorem 3.3. *Let β be a proximity base on a set X and let $F, G \in S(X, E)$. Define a binary relation as: $(F, G) \notin \beta^i$ if and only if there exist subsets A, B of X such that $F \sqsubseteq \tilde{A}$, $G \sqsubseteq \tilde{B}$ and $(A, B) \notin \beta$. Then, β^i is a soft proximity base on X which is induced by proximity base β. (Here, for every $A \subseteq X$, $\tilde{A}$ is the soft set over X defined by $\tilde{A}(e) = A$ for all $e \in E$.)*

Proof. We show that β is a soft proximity base. For this, it suffices to verify axioms given in the definition of soft proximity base.
(i) Obviously, $(\Phi, F) \notin \beta^i$.

(ii) Let $(F, G) \notin \beta^i$. Then there exist subsets A, B of X such that $F \sqsubseteq \tilde{A}$, $G \sqsubseteq \tilde{B}$ and $(A, B) \notin \beta$. Therefore $A \cap B = \varnothing$ as β is a proximity base and thus, $\tilde{A} \sqcap \tilde{B} = \Phi$ which implies $F \sqcap G = \Phi$.

(iii) Symmetry is obvious.

(iv) Let $(F^*, G^*) \notin \beta^i$ and $F \sqsubseteq F^*$, $G \sqsubseteq G^*$. Then there exist subsets A, B of X such that $F^* \sqsubseteq \tilde{A}$, $G^* \sqsubseteq \tilde{B}$ and $(A, B) \notin \beta$. Thus, we have $F \sqsubseteq \tilde{A}$, $G \sqsubseteq \tilde{B}$ and $(A, B) \notin \beta$. Hence, $(F, G) \notin \beta^i$.

(v) Let $(F, G) \notin \beta^i$. Then there exist subsets A, B of X such that $F \sqsubseteq \tilde{A}$, $G \sqsubseteq \tilde{B}$ and $(A, B) \notin \beta$. Since $(A, B) \notin \beta$ therefore, there exists a subset C of X such that $(A, C) \notin \beta$ and $(X \smallsetminus C, B) \notin \beta$. Thus, there exist a soft set $\tilde{C}$ corresponding to C such that $(F, \tilde{C}) \notin \beta^i$ and $(\tilde{X} \smallsetminus \tilde{C}, G) \notin \beta^i$. □

Theorem 3.4. *Let β_0 be a soft proximity base on X. Then the following two statements are equivalent:*
(i) There exists a proximity base β on X such that $\beta_0 = \beta^i$.

(ii) If $(F, G) \notin \beta_0$, then there exist subsets A, B of X such that $F \sqsubseteq \tilde{A}$, $G \sqsubseteq \tilde{B}$ and $(\tilde{A}, \tilde{B}) \notin \beta_0$.

Proof. Obviously, $(i) \Rightarrow (ii)$.

$(ii) \Rightarrow (i)$ Define a binary relation β on $\mathcal{P}(X)$ as $(A, B) \in \beta$ if and only if $(\tilde{A}, \tilde{B}) \in \beta_0$. We claim that β is a proximity base. Clearly, $B(i)$-$B(iv)$ can be easily verified. Now for $B(v)$, assume that $(A, B) \notin \beta$ which implies $(\tilde{A}, \tilde{B}) \notin \beta_0$. Therefore, there exist a soft set H in $S(X, E)$ such that $(\tilde{A}, H) \notin \beta_0$ and $(\tilde{X} \smallsetminus H, \tilde{B}) \notin \beta_0$. So, by (2), there are subsets E, F, G, L of X such that $\tilde{A} \sqsubseteq \tilde{E}$, $H \sqsubseteq \tilde{F}$, $\tilde{B} \sqsubseteq \tilde{G}$, $\tilde{X} \smallsetminus H \sqsubseteq \tilde{L}$ and $(\tilde{E}, \tilde{F}) \notin \beta_0$, $(\tilde{G}, \tilde{L}) \notin \beta_0$. Now, since $H \sqsubseteq \tilde{F}$, $\tilde{X} \smallsetminus H \sqsubseteq \tilde{L}$ and $(\tilde{E}, \tilde{F}) \notin \beta_0$ so we have $X \smallsetminus F \subseteq L$. Thus, we have $(\tilde{A}, \tilde{F}) \notin \beta_0$ because $(\tilde{E}, \tilde{F}) \notin \beta_0$ and $\tilde{A} \sqsubseteq \tilde{E}$. Hence, $(A, F) \notin \beta$. Similarly, we can easily show that $(\tilde{X} \smallsetminus \tilde{F}, \tilde{B}) \notin \beta_0$ which implies $(X \smallsetminus F, B) \notin \beta$. Now it only remains to show that $\beta_0 = \beta^i$. Firstly, suppose that $(F, G) \notin \beta_0$ then, by hypothesis there exist subsets A, B of X such that $F \sqsubseteq \tilde{A}$, $G \sqsubseteq \tilde{B}$ and $(\tilde{A}, \tilde{B}) \notin \beta_0$. Therefore, there are subsets A, B of X such that $F \sqsubseteq \tilde{A}$, $G \sqsubseteq \tilde{B}$ and $(A, B) \notin \beta$ which shows that $(F, G) \notin \beta^i$. Conversely, assume that $(F, G) \notin \beta^i$. Then there exist subsets A, B, of X such that $F \sqsubseteq \tilde{A}$, $G \sqsubseteq \tilde{B}$ and $(A, B) \notin \beta$ and hence $(\tilde{A}, \tilde{B}) \notin \beta_0$ which implies that $(F, G) \notin \beta_0$. □

Corollary 3.5. *Let β_0 be a soft proximity base on X and there exist a proximity base β such that $\beta_0 = \beta^i$. Then the relation $(A, B) \in \beta$ holds if and only if $(\tilde{A}, \tilde{B}) \in \beta_0$ is a proximity base on X.*

In the next theorem, we generate a soft proximity from a given soft proximity base and obtain an algorithm of finding it in the proof.

Theorem 3.6. *Let β be a soft proximity base on a set X. For any F, G in $S(X, E)$, define a binary relation as: $(F, G) \in \delta(\beta)$ if and only if given any finite soft covers $\{F_i : i \in J_m\}$ and $\{G_j : j \in J_n\}$ of F and G, respectively, there exist $(i, j) \in J_m \times J_n$ such that $(F_i, G_j) \in \beta$. Then $\delta(\beta)$ is the coarsest soft proximity finer than the relation β. Moreover, $\delta(\beta)$ is separated if and only if the soft proximity base β is separated. (Here J_m is a set of first m natural numbers and $\delta(\beta)$ is a soft proximity generated by the soft proximity base β.)*

Proof. We first show that $\delta(\beta)$ is a soft proximity. Clearly, $\delta(\beta)$ satisfies SP(i)–SP(iii) trivially. Now, if $(F, G) \in \delta(\beta)$, then since any finite soft cover of $G \sqcup H$ is also a finite soft cover of G therefore, $(F, G \sqcup H) \in \delta(\beta)$. For necessary part, assume $(F, G) \notin \delta(\beta)$ and $(F, H) \notin \delta(\beta)$. We show that $(F, G \sqcup H) \notin \delta(\beta)$. Since $(F, G) \notin \delta(\beta)$. Therefore, there exist a finite soft cover $\{F_i : i \in J_m\}$ and $\{G_j : j \in J_n\}$ of F and G, respectively, such that $(F_i, G_j) \notin \beta$ for all $(i, j) \in J_m \times J_n$. Similarly, as $(F, H) \notin \delta(\beta)$. Therefore, there exist finite soft cover $\{K_l : l \in J_p\}$ and $\{W_c : c \in J_q\}$ of F and H, respectively, such that $(K_l, W_c) \notin \beta$ for all $(l, c) \in J_p \times J_q$. Now put $G_{n+1} = W_1$, $G_{n+2} = W_2, \cdots$, $G_{n+q} = W_q$ and $S_{(i,l)} = F_i \sqcap K_l$, then $\{G_j : j \in J_{n+q}\}$ and $\{S_{(i,l)} : (i, l) \in J_m \times J_p\}$ are soft cover of $G \sqcup H$ and F, respectively. By the axiom SB(iv) of soft proximity base β and the above construction, we conclude that $(S_{(i,l)}, G_j) \notin \beta$ for all $(i, l) \in J_m \times J_p$ and $j \in J_{n+q}$ which implies $(F, G \sqcup H) \notin \delta(\beta)$. Thus, $\delta(\beta)$ satisfies $SP(iv)$ axiom also. Now let $(F, G) \notin \delta(\beta)$ therefore, there exist a finite soft cover $\{F_i : i \in J_m\}$ and $\{G_j : j \in J_n\}$ of F and G, respectively, such that $(F_i, G_j) \notin \beta$ for all $(i, j) \in J_m \times J_n$. Since $(F_i, G_j) \notin \beta$ for each $(i, j) \in J_m \times J_n$ and as β is a soft proximity base, therefore, there exist H_{ij} in $S(X, E)$ for each $(i, j) \in J_m \times J_n$ such that $(F_i, H_{ij}) \notin \beta$ and $(\tilde{X} \setminus H_{ij}, G_j) \notin \beta$. Now put $H_j = \bigcap_{i \in J_m} H_{ij}$ and $H = \bigcup_{j \in J_n} H_j$. Therefore $(F_i, H_j) \notin \beta$ for any $(i, j) \in J_m \times J_n$. Thus, by definition of $\delta(\beta)$, $(F, H) \notin \delta(\beta)$. Similarly, we get $(\tilde{X} \setminus H, G) \notin \delta(\beta)$. Hence, $\delta(\beta)$ is a soft proximity on (X, E).

Clearly, $\delta(\beta) \geq \beta$. Now it only remains to show that $\delta(\beta)$ is the coarsest soft proximity on X such that $\delta(\beta) \geq \beta$. Suppose δ be an arbitrary soft proximity such that $\delta \geq \beta$. Let $(F, G) \in \delta$. If $\{F_i : i \in J_m\}$ and $\{G_j : j \in J_n\}$ are any finite soft covers of F and G, respectively, then there exist $(i, j) \in J_m \times J_n$ such that $(F_i, G_j) \in \delta$ which implies $(F_i, G_j) \in \beta$ because $\delta \geq \beta$. Thus, by definition of $\delta(\beta)$ we have $(F, G) \in \delta(\beta)$ for all (F, G). Hence $\delta \geq \delta(\beta)$. It is obvious that $\delta(\beta)$ is separated if and only if β is separated. □

Next, we obtain a relation between induced soft proximity $\delta^i(\beta)$ from the proximity $\delta(\beta)$ generated by proximity base β [25] and the soft proximity $\delta(\beta^i)$ generated by induced soft proximity base β^i as shown in the following theorem.

Theorem 3.7. *Let β be a proximity base on X. Then $\delta^i(\beta) > \delta(\beta^i)$.*

Proof. Since, in view of Theorem 3.6, $\delta(\beta^i)$ is the coarsest soft proximity finer than β^i, it suffices to show that $\delta^i(\beta) > \beta^i$. Now, let $(F, G) \in \delta^i(\beta)$; then, for any subsets A, B of X such that $F \sqsubseteq \tilde{A}$, $G \sqsubseteq \tilde{B}$ we have $(A, B) \in \delta(\beta)$. As $\delta(\beta) > \beta$. Therefore, we have $(A, B) \in \beta$ which implies $(F, G) \in \beta^i$. □

DEFINITION 3.8. Let β be a soft proximity base on X and $\delta(\beta)$ is a soft proximity generated by β. For $F, G \in S(X, E)$, the soft set G is called soft neighbourhood of F with respect to β if $(F, \tilde{X} \setminus G) \notin \beta$. We write this in symbols as $F \Subset_\beta G$.

The following theorem can be easily proved by using the definition of soft neighbourhood with respect to β.

THEOREM 3.9. *Let β be a soft proximity base on X and $\delta(\beta)$ be a soft proximity generated by β. Then the relation $\Subset_\beta$ satisfies the following properties:*
(i) $\Phi \Subset_\beta F$. *(ii) If $F \Subset_\beta G$, then $(\tilde{X} \setminus G) \Subset_\beta (\tilde{X} \setminus F)$.*
(iii) If $F \Subset_\beta G$, then $F \sqsubseteq G$. *(iv) If $F \Subset_\beta (G \sqcap H)$, then $F \Subset_\beta G$ and $F \Subset_\beta H$.*
(v) If $F_1 \sqsubseteq F \Subset_\beta G \sqsubseteq G_1$, then $F_1 \Subset_\beta G_1$.
(vi) If $F \Subset_\beta G$, then there exist an H in $S(X, E)$ such that $F \Subset_\beta H \Subset_\beta G$.

THEOREM 3.10. *Suppose $\Subset_\beta$ be a relation on $S(X, E)$ which satisfies all the properties of the previous Theorem 3.9. Then β is a soft proximity base on X which can be defined for all F, G in $S(X, E)$ as $(F, G) \notin \beta$ if and only if $F \Subset_\beta \tilde{X} \setminus G$.*

Proof. For proving β to be a soft proximity base, we just need to verify axioms SB(i)–SB(v). Clearly SB(i)–SB(iii) hold. Now, for SB(iv) assume that $(F^*, G^*) \notin \beta$ and $F \sqsubseteq F^*$, $G \sqsubseteq G^*$. Then, $F^* \Subset_\beta \tilde{X} \setminus G^*$ and $\tilde{X} \setminus G^* \sqsubseteq \tilde{X} \setminus G$. Therefore, by property (v) of Theorem 3.9, we have $F \Subset_\beta \tilde{X} \setminus G$ which implies $(F, G) \notin \beta$. For the last axiom, let $(F, G) \notin \beta$. Then $F \Subset_\beta \tilde{X} \setminus G$. Thus, by property (iv) of Theorem 3.9, there exists an H in $S(X, E)$ such that $F \Subset_\beta H \Subset_\beta \tilde{X} \setminus G$. Hence, $(F, \tilde{X} \setminus H) \notin \beta$ and $(H, G) \notin \beta$. □

We deduce from the binary relation β defined in above theorem that G is a soft neighbourhood of F with respect to β if and only if $F \Subset_\beta G$. Further, by using soft set, we generalize the notion of proximity subbase [25] to soft proximity subbase whose definition is given below.

DEFINITION 3.11. Let X be a non-empty set. A soft proximity subbase on X is a binary relation s on $S(X, E)$ satisfying the following two axioms for all $F, G, H \in S(X, E)$:
SS(i) If $F \sqcap G \neq \Phi$, then $(F, G) \in s$;

SS(ii) If $(F, G) \notin s$, then there exists an $H \in S(X, E)$ such that $(F, H) \notin s$ and $(\tilde{X} \setminus H, G) \notin s$.

DEFINITION 3.12. A soft proximity subbase s on X is called separated if it satisfies the following axiom:
If x^{e_1}, y^{e_2} are two distinct soft elements of (X, E) and $(\{x^{e_1}\}, \{y^{e_2}\}) \in s$, then there exist two soft subsets P and Q such that $x^{e_1} \tilde{\in} P$, $y^{e_2} \tilde{\in} Q$ and either $(P, Q) \notin s$ or $(Q, P) \notin s$.

When a proximity base is given, there always exists a soft proximity base. The similar result hold for proximity subbase and soft proximity subbase.

THEOREM 3.13. *Let s be a proximity subbase on a set X. Let $F, G \in S(X,E)$ define a binary relation as: $(F,G) \notin s^i$ if and only if there exist subsets A, B of X such that $F \sqsubseteq \tilde{A}$, $G \sqsubseteq \tilde{B}$ and $(A,B) \notin s$. Then s^i is a soft proximity subbase on X which is induced by proximity subbase s. (Here, for every $A \subseteq X$, $\tilde{A}$ is the soft set over X defined by $\tilde{A}(e) = A$ for all $e \in E$.)*

THEOREM 3.14. *If s is a soft proximity subbase on X, then there exists the coarsest soft proximity $\delta(s)$ on X finer than the relation s. Moreover, $\delta(s)$ is separated if and only if s is separated. (In fact, $\beta(s)$ is the coarsest soft proximity base finer than s and generated by s.)*

Proof. For any F, G in $S(X,E)$, define a binary relation $\beta(s)$ on $S(X,E)$ as follows: $(F,G) \in \beta(s)$ if and only if $F \neq \Phi$, $G \neq \Phi$ and for any soft sets F^*, G^* such that $F \sqsubseteq F^*$, $G \sqsubseteq G^*$, both (F^*,G^*) and (G^*,F^*) are elements of s.

Clearly, by definition, $\beta(s)$ satisfies SB(i)–SB(iii). Now suppose that $(F,G) \in \beta(s)$ and $F \sqsubseteq F^{**}$, $G \sqsubseteq G^{**}$. We show that $(F^{**},G^{**}) \in \beta(s)$. Since $(F,G) \in \beta(s)$ therefore, $F \neq \Phi$, $G \neq \Phi$ which implies $F^{**} \neq \Phi$, $G^{**} \neq \Phi$. Let F^*, G^* be two soft sets such that $F^{**} \sqsubseteq F^*$, $G^{**} \sqsubseteq G^*$, then as $(F,G) \in \beta(s)$ so by hypothesis both (F^*,G^*) and (G^*,F^*) are elements of s. Thus $(F^{**},G^{**}) \in \beta(s)$. Hence SB(iv) hold for $\beta(s)$. To prove that $\beta(s)$ satisfies SB(v), assume that $(F,G) \notin \beta(s)$, then the following two cases occur:

Case 1. If $F = \Phi$ and $G \neq \Phi$, then take $H = \tilde{X}$ we have $(F,H) = (\Phi,\tilde{X}) \notin \beta(s)$ and $(\tilde{X} \setminus H, G) = (\Phi, G) \notin \beta(s)$. Thus SB(v) is satisfied. Similarly, when $G = \Phi$ and $F \neq \Phi$, then $H = \Phi$ clinches the matter.

Case 2. Suppose $F \neq \Phi$, $G \neq \Phi$, then since $(F,G) \notin \beta(s)$ therefore, there exist F^*, G^* such that $F \sqsubseteq F^*$, $G \sqsubseteq G^*$ and either $(F^*,G^*) \notin s$ or $(G^*,F^*) \notin s$. If $(F^*,G^*) \notin s$ then, by SS(ii), there exists H in $S(X,E)$ such that $(F^*,H) \notin s$ and $(\tilde{X} \setminus H, G^*) \notin s$ thus, $(F,H) \notin \beta(s)$. Similarly, $(\tilde{X} \setminus H, G) \notin \beta(s)$. If $(G^*,F^*) \notin s$ then, by similar argument, there exists H in $S(X,E)$ such that $(G,H) \notin \beta(s)$ and $(\tilde{X} \setminus H, F) \notin \beta(s)$. Hence, $\beta(s)$ is soft proximity base on X. Obviously, $\beta(s) \geq s$. Using definition of $\beta(s)$, it can be easily shown that $\beta(s)$ is the coarsest soft proximity base finer than s. Now let δ be any soft proximity such that $\delta \geq s$ then $\delta \geq \beta(s)$ and since $\delta(s)$ is the coarsest soft proximity finer than $\beta(s)$. So, we have $\delta \geq \delta(s)$ that is, $\delta(s)$ is the coarsest soft proximity finer than s. It can be easily shown that s is separated if and only if $\beta(s)$ is separated and we know that $\delta(s)$ is separated if and only if $\beta(s)$ is separated. Hence the theorem is proved. □

THEOREM 3.15. *Let $\{\delta_a : a \in I\}$ be a non-empty collection of soft proximities on X. Then there exists a coarsest soft proximity δ on X such that $\delta > \delta_a$ for all $a \in I$. (The coarsest soft proximity δ is denoted by $\sup\{\delta_a : a \in I\}$.)*

Proof. Let $\beta = \bigcap\{\delta_a : a \in I\}$; then β is a soft proximity base. Hence $\delta(\beta)$ is the required coarsest soft proximity such that $\delta(\beta) > \delta_a$ for all $a \in I$. □

Corollary 3.16. *Let $\{\delta_a : a \in I\}$ be a non-empty collection of soft proximities on X. Then $\mathcal{T}[\sup\{\delta_a : a \in I\}] = \sup\{\mathcal{T}(\delta_a) : a \in I\}$.*

Proof. Since $\sup\{\delta_a : a \in I\} > \delta_a$ for each a in I. Therefore $\mathcal{T}(\delta_a) \subseteq \mathcal{T}[\sup\{\delta_a : a \in I\}]$ for each a in I. Thus, $\sup\{\mathcal{T}(\delta_a) : a \in I\} \subseteq \mathcal{T}[\sup\{\delta_a : a \in I\}]$ and the other inclusion holds by the fact that the finest soft proximity δ compatible with $\sup\{\mathcal{T}(\delta_a) : a \in I\}$ is finer than δ_a for each a in I. Therefore, we have $\delta \geq \sup\{\delta_a : a \in I\}$. Hence the other inclusion $\sup\{\mathcal{T}(\delta_a) : a \in I\} \supseteq \mathcal{T}[\sup\{\delta_a : a \in I\}]$ holds. □

Theorem 3.17. *Let $\{\delta_a : a \in I\}$ be a non-empty collection of soft proximities on X. Then there exists a finest soft proximity δ on X such that δ is coarser than δ_a for each a in I. (The finest soft proximity δ is denoted by $\inf\{\delta_a : a \in I\}$.)*

Proof. Let $\mathcal{A}$ be the collection of soft proximities defined as: $\mathcal{A} = \{\delta_p : \delta_a \geq \delta_p$ for each $a \in I\}$. Obviously, the collection $\mathcal{A}$ is non-empty as the indiscrete soft proximity on X is a member of it. Let $\delta = \sup\{\delta_p : \delta_p \in \mathcal{A}\}$. Here we show that $\delta_a \geq \delta$ for each a in I. Suppose $a \in I$ is arbitrary and $(F,G) \in \delta_a$. If $\{F_i : i \in J_m\}$ and $\{G_j : j \in J_n\}$ are finite soft covers of F and G respectively, then there exist $(i,j) \in J_m \times J_n$ such that $(F_i, G_j) \in \delta_a$. Thus, for the same pair (i,j), $(F_i, G_j) \in \delta_p$ for each $\delta_p \in \mathcal{A}$. Therefore $(F_i, G_j) \in \beta$ where $\beta = \bigcap\{\delta_p : \delta_p \in \mathcal{A}\}$ and as β is a soft proximity base for δ. We have $(F,G) \in \delta$. Hence $\delta_a \geq \delta$. As a was arbitrary, it is true for each $a \in I$. Clearly δ is finer than each member of $\mathcal{A}$. So δ is the finest soft proximity on X and it is coarser than each member of the collection $\{\delta_a : a \in I\}$. □

Theorem 3.18. *The collection of all soft proximities on a non-empty set X forms a complete lattice under the ordering $\geq$.*

Now, in the next theorem and corollary, we give a necessary and sufficient condition for being a soft p-continuous map in terms of soft proximity subbase and base, respectively.

Theorem 3.19. *Let (X, δ_1, E) and (Y, δ_2, K) be two soft proximity spaces and let s be a soft proximity subbase for the soft proximity δ_2. A soft map $\varphi_\psi : (X, \delta_1, E) \longrightarrow (Y, \delta_2, K)$ is soft p-continuous if and only if $(F,G) \notin s$ implies $(\varphi_\psi^{-1}(F), \varphi_\psi^{-1}(G)) \notin \delta_1$ for all F, G in $S(Y,K)$.*

Proof. Let φ_ψ be soft p-continuous map and $(F,G) \notin s$, then $(F,G) \notin \delta_2$ implies $(\varphi_\psi^{-1}(F), \varphi_\psi^{-1}(G)) \notin \delta_1$ for all F, G in $S(Y,K)$ because φ_ψ is soft p-continuous.

Conversely, assume that $(F,G) \notin s$ implies $(\varphi_\psi^{-1}(F), \varphi_\psi^{-1}G) \notin \delta_1$ for all F, G in $S(Y,K)$. Suppose $\beta(s)$ be the soft proximity base on Y generated by soft proximity subbase s. Let $(F,G) \notin \delta_2$. Now, we show that $(\varphi_\psi^{-1}(F), \varphi_\psi^{-1}(G)) \notin \delta_1$. If $F = \Phi$ or $G = \Phi$ or $(F,G) \notin s$ then we are done. And if there are two soft sets, F^* and G^* such that $F \sqsubseteq F^*$, $G \sqsubseteq G^*$ and either $(F^*, G^*) \notin s$ or $(G^*, F^*) \notin s$ then also we are done. Hence, we have proved that if $(F,G) \notin \beta(s)$, then $(\varphi_\psi^{-1}(F), \varphi_\psi^{-1}(G)) \notin \delta_1$. Now suppose $(F,G) \in \beta(s) - \delta_2$. Then there exist finite soft covers $\{F_i : i \in J_m\}$ and $\{G_j : j \in J_n\}$ of F and G respectively, such that $(F_i, G_j) \notin \beta(s)$ for all $(i,j) \in J_m \times J_n$ and therefore $(\bigsqcup_1^m \varphi_\psi^{-1}(F_i), \bigsqcup_1^n \varphi_\psi^{-1}(G_j)) \notin \delta_1$ for all $(i,j) \in J_m \times J_n$. Hence, $(\varphi_\psi^{-1}(F), \varphi_\psi^{-1}(G)) \notin \delta_1$, which completes the proof. □

Corollary 3.20. *Let* (X,δ_1,E) *and* (Y,δ_2,K) *be two soft proximity spaces and let* β *be a soft proximity base for the soft proximity* δ_2*. A soft map* $\varphi_\psi : (X,\delta_1,E) \longrightarrow (Y,\delta_2,K)$ *is soft p-continuous if and only if* $(F,G) \notin \beta$ *implies* $(\varphi_\psi^{-1}(F),\varphi_\psi^{-1}(G)) \notin \delta_1$ *for all* F, G *in* $S(Y,K)$.

4. Induced soft proximities

Applications of soft proximity bases and subbases are numerous, reason being, quite a few theorems in soft proximity space can effortlessly be substantiated by employing soft proximity base and subbase.

Theorem 4.1. *Let* X *be a non-empty set. Let* $\mathcal{F}$ *be a non-empty family of soft maps such that each member* φ_ψ *of* $\mathcal{F}$ *is a map from* (X,E) *into a soft proximity space* $(Y_{\varphi_\psi},\delta_{\varphi_\psi},K_{\varphi_\psi})$*. Then there exists a coarsest soft proximity on* X *such that each member of* $\mathcal{F}$ *is soft p-continuous.*

Proof. Firstly, for any F, G in $S(X,E)$, define a binary relation β on $S(X,E)$ as follows: $(F,G) \in \beta$ if and only if $(\varphi_\psi(F),\varphi_\psi(G)) \in \delta_{\varphi_\psi}$ for each member φ_ψ of $\mathcal{F}$.

We prove β is a soft proximity base on X. Clearly, axioms SB(i)–SB(iv) can be easily verified. Now for SB(v) assume that $(F,G) \notin \beta$, then there exist a member φ_ψ of $\mathcal{F}$ such that $(\varphi_\psi(F),\varphi_\psi(G)) \notin \delta_{\varphi_\psi}$. Therefore by $SP(v)$, there exist H_{φ_ψ} such that $(\varphi_\psi(F),H_{\varphi_\psi}) \notin \delta_{\varphi_\psi}$ and $(\tilde{Y}_{\varphi_\psi} \setminus H_{\varphi_\psi},\varphi_\psi(G)) \notin \delta_{\varphi_\psi}$. Let $H = \varphi_\psi^{-1}(H_{\varphi_\psi})$. Thus, it can be easily verified that $(F,H) \notin \beta$ and $(\tilde{X} \setminus H,G) \notin \beta$. Hence β is a soft proximity base. Therefore $\delta(\beta)$ is the coarsest soft proximity on X finer than β. Obviously each φ_ψ is soft p-continuous by the definition of β. Hence $\delta(\beta)$ is the required soft proximity. □

The above theorem can also be generalized just by changing the soft proximity δ_{φ_ψ} to soft proximity base β_{φ_ψ} of it for any φ_ψ of $\mathcal{F}$. In fact, we can say more than this in the following theorem.

Theorem 4.2. *Let* X *be a non-empty set and let* $\mathcal{F}$ *be a non-empty family of soft maps such that each member* φ_ψ *of* $\mathcal{F}$ *is a map from* (X,E) *onto a soft proximity space* $(Y_{\varphi_\psi},\delta_{\varphi_\psi},K_{\varphi_\psi})$*. For each* $\varphi_\psi \in \mathcal{F}$*, let* $\tilde{S}_{\varphi_\psi}$ *be a soft proximity subbase for* δ_{φ_ψ}*. Then there exists a soft proximity subbase* $\tilde{S}$ *such that the coarsest soft proximity* $\delta(S)$ *on* X *is the soft proximity which makes each* φ_ψ *a soft p-continuous map.*

Proof. For any F, G in $S(X,E)$, define a binary relation $\tilde{S}$ as follows: $(F,G) \in \tilde{S}$ if and only if $(\varphi_\psi(F),\varphi_\psi(G)) \in \tilde{S}_{\varphi_\psi}$ for each φ_ψ in $\mathcal{F}$. Then $\tilde{S}$ is the required soft proximity subbase. □

In the next theorem, which is a direct consequence of Theorem 4.1, we give a soft proximity base for the product soft proximity and conclude that product soft proximity can be characterized in terms of soft proximity base.

Theorem 4.3. *Let* $\{(X_a, \delta_a, E_a) : a \in I\}$ *be a non-empty family of soft proximity spaces, and let* $X = \prod_{a\in I} X_a$ *and* $E = \prod_{a\in I} E_a$ *be sets. For each* $a \in I$*, let* $(p_{X_a})_{q_{E_a}}$ *be a soft mapping. Let* F*,* G *in* $S(X,E)$*, then define a binary relation* β *on* $S(X,E)$ *as follows:* $(F,G) \in \beta$ *if and only if* $((p_{X_a})_{q_{E_a}}(F), (p_{X_a})_{q_{E_a}}(G)) \in \delta_a$ *for every* a *in* I*. Then* β *is a soft proximity base on* X *for the product soft proximity.*

Theorem 4.4. *Let* $\{(X_a, \delta_a, E_a) : a \in I\}$ *be a non-empty family of soft proximity spaces and let* (X, δ, E) *be the product soft proximity space. A map* φ_ψ *on* (Y, δ^*, K) *to the product* (X, δ, E) *is soft* p*-continuous if and only if the composition* $(p_{X_a})_{q_{E_a}} \circ \varphi_\psi$ *is soft* p*-continuous for each* a *in* I*.*

Proof. Necessary part follows from the result that the composition of two soft p-continuous maps is soft p-continuous. For the sufficient part, let $(p_{X_a})_{q_{E_a}} \circ \varphi_\psi$ is soft p-continuous for each a in I. Put $(p_{X_a})_{q_{E_a}} \circ \varphi_\psi = \Gamma_a$. Now, we show that φ_ψ is soft p-continuous. Suppose β be the soft proximity base on X defined by $(F,G) \in \beta$ if and only if $((p_{X_a})_{q_{E_a}}(F), (p_{X_a})_{q_{E_a}}(G)) \in \delta_a$ for every a in I. Let $(F,G) \notin \beta$, then there exist some a in I such that $((p_{X_a})_{q_{E_a}}(F), (p_{X_a})_{q_{E_a}}(G)) \notin \delta_a$. Thus by soft p-continuity of Γ_a, we have $(\Gamma_a^{-1}(p_{X_a})_{q_{E_a}}(F), \Gamma_a^{-1}(p_{X_a})_{q_{E_a}}(G)) \notin \delta$. Since $\Gamma_a^{-1} = \varphi_\psi^{-1} \circ (p_{X_a})_{q_{E_a}}^{-1}$ and $F \sqsubseteq (p_{X_a})_{q_{E_a}}^{-1}((p_{X_a})_{q_{E_a}}(F))$, $G \sqsubseteq (p_{X_a})_{q_{E_a}}^{-1}((p_{X_a})_{q_{E_a}}(G))$. Therefore we have $(\varphi_\psi^{-1}(F), \varphi_\psi^{-1}(G)) \in \delta$. Hence by Corollary 3.20, we conclude that φ_ψ is soft p-continuous. □

5. Conclusion

Proximity is a exquisite structure than topology and provides a lucid and conceptual approach to many substantial topological problems. So many authors studied it for the progress in topology and also worked on proximity in fuzzy and soft contexts as well. In the current work, we introduce soft proximity bases and subbases, and identify the relation between proximity base and soft proximity base. Multiple theorems in soft proximity spaces have remarkably been simplified by applying soft proximity bases (proximity subbases). For example, soft p-continuity and product soft proximity are characterized in terms of soft proximity bases (subbases). We anticipate that the findings in this paper will benefit researcher to augment and stimulate the further study with regard to soft proximity spaces.

Acknowledgement. The second author is financially supported by University Grants Commission, India.

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(received 10.05.2019; in revised form 06.12.2020; available online 11.04.2021)

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