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SOME REMARKS ON TAC-CONTRACTIVE MAPPINGS IN b-METRIC SPACES

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Abstract. In this paper we complement and improve fixed point results established on TAC-contractive mappings in the framework of b-metric spaces. Our analysis starts by a recent paper of Babu and Dula [G. V. R. Babu and T. M. Dula, Fixed points of generalized TAC-contractive mappings in b-metric spaces, Matematički Vesnik 69 (2017), no. 2, 75–88].

1. Introduction and preliminaries

In 2016 Chandok et al. introduced in [4] the notion of TAC-contractive mapping as follows.

Definition 1.1. Let (X, d) be a metric space and let $\alpha, \beta : X \to [0, +\infty)$ be two given mappings. A mapping $T : X \to X$ is called a TAC-contractive mapping if for each $x, y \in X$ such that $\alpha(x)\beta(y) \geq 1$ one has

$$\psi(d(Tx, Ty)) \leq f(\psi(d(x, y)), \phi(d(x, y)))$$

where ψ, ϕ and f are functions that verify the following conditions:

(i) $\psi : [0, +\infty) \to [0, +\infty)$ is a nondecreasing continuous function such that $\psi(t) = 0$ if and only if $t = 0$;

(ii) $\phi : [0, +\infty) \to [0, +\infty)$ is a continuous function such that $\lim_{n\to\infty} \phi(t_n) = 0$ implies $\lim_{n\to\infty} t_n = 0$;

(iii) $f : [0, +\infty)^2 \to \mathbb{R}$ is a continuous function such that $f(s, t) \leq s$. Further, from $f(s, t) = s$ it follows that for all $s, t \in [0, +\infty)$ either $s = 0$ or $t = 0$.

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In the sequel, we will denote by Ψ the set of functions that satisfy condition (i) of Definition 1.1. Following [12], we will call Ψ the set of altering distance functions. Moreover, we will denote by Φ the set of functions $\phi : [0, +\infty) \to [0, +\infty)$ satisfying (ii) of Definition 1.1 and we will write $\mathcal{F}$ for the set of functions $f : [0, \infty)^2 \to \mathbb{R}$ satisfying Definition 1.1 (iii).

Definition 1.2. ([13]) Let A and B be two nonempty subsets of a set X. A mapping $T : A \cup B \to A \cup B$ is called cyclic if $T(A) \subset B$ and $T(B) \subset A$.

Definition 1.3. ([1]) Let X be a nonempty set and let $\alpha, \beta : X \to [0, +\infty)$ be two given mappings. A mapping $T : X \to X$ is called a cyclic (α, β)-admissible mapping if the following conditions are satisfied:

(i) for any $x \in X$ such that $\alpha(x) \geq 1$ one has $\beta(Tx) \geq 1$;

(ii) for any $y \in X$ such that $\beta(y) \geq 1$ one has $\alpha(Ty) \geq 1$.

In [4], Chandok et al. proved some fixed point results in the framework of complete metric spaces. One among of this results is the following theorem.

Theorem 1.4. *Let (X, d) be a complete metric space, let $\alpha, \beta : X \to [0, +\infty)$ be two given mappings and let $T : X \to X$ be a cyclic (α, β)-admissible mapping. Assume that T is a TAC-contractive mapping and assume that following condition is satisfied:*

(a_1) there exists $x_0 \in X$ such that $\alpha(x_0) \geq 1$ and $\beta(x_0) \geq 1$.

If in addition also one of the following conditions is satisfied

(a_2) T is continuous,

(a_3) if $\{x_n\}$ is a sequence in X such that $x_n \to x$ and $\beta(x_n) \geq 1$ for all n one has also $\beta(x) \geq 1$,

then T has a fixed point. Furthermore, if $\alpha(u) \geq 1$ and $\beta(v) \geq 1$ for all $u, v \in Fix(T)$, where $Fix(T)$ is the set of fixed points of T, then T has a unique fixed point.

The notion of TAC-contractive mapping was extended to b-metric spaces by Babu and Dula in [2]. Let X a nonempty set. We recall that a mapping $d : X \times X \to [0, +\infty)$ is called a b-metric if it satisfies the following conditions:

(b_1) $d(x, y) = 0$ if and only if $x = y$;

(b_2) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(b_3) there exists $s \geq 1$ such that $d(x, z) \leq s[d(x, y) + d(y, z)]$ for all $x, y, z \in X$.

The pair (X, d) is called a b-metric space with coefficient s. We refer to [2, 3, 5–11, 14, 15] and the references therein for notions as b-convergence, b-completeness and b-Cauchy sequence in b-metric spaces.

We stress that any metric space is a b-metric space with coefficient $s = 1$. Generally, a b-metric space is not a metric space. We observe for example that if $\mathbb{R}$ denotes the real line and $X = \mathbb{R}$ then the mapping $d : X \times X \to [0, +\infty)$ defined by $d(x, y) = (x - y)^2$ is a b-metric on the set $X = \mathbb{R}$ but (X, d) is not a metric space.

In [2], Babu and Dula introduced the notions of generalized TAC-contractive mapping and generalized TAC-cyclic contractive mapping as follows.

Definition 1.5. Let (X, d) be a b-metric space with coefficient $s \geq 1$ and let $\alpha, \beta : X \to [0, +\infty)$ be two given mappings. A mapping $T : X \to X$ is called a generalized TAC-contractive mapping if there exist $\psi \in \Psi, \phi \in \Phi$ and $f \in \mathcal{F}$ such that for all $x, y \in X$ with $\alpha(x)\beta(y) \geq 1$ one has $\psi\left(s^3 d(Tx, Ty)\right) \leq f\left(\psi(M_s(x, y)), \phi(M_s(x, y))\right)$, where $M_s(x, y) = \max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x,Ty)+d(y,Tx)}{2s}\right\}$.

Definition 1.6. Let (X, d) be a b-metric space with coefficient $s \geq 1$ and let A and B be two closed subsets of X such that $A \cap B \neq \emptyset$. A cyclic mapping $T : A \cup B \to A \cup B$ is called a generalized TAC-cyclic contractive mapping if there exist $\psi \in \Psi, \phi \in \Phi$ and $f \in \mathcal{F}$ such that for all $x \in A$ and $y \in B$ the following condition is satisfied $\psi\left(s^3 d(Tx, Ty)\right) \leq f\left(\psi(d(x, y)), \phi(d(x, y))\right)$.

Furthermore, Babu and Dula extended the results of Theorem 1.4 in the framework of b-metric spaces. Precisely, they proved the following fixed point results.

Theorem 1.7. *([2, Theorem 3.2]) Let (X, d) be a b-complete b-metric space with coefficient $s \geq 1$ and let $T : X \to X$. Assume there exist mappings $\alpha, \beta : X \to [0, +\infty)$, $\psi \in \Psi$, $\phi \in \Phi$ and $f \in \mathcal{F}$ such that T is a generalized TAC-contractive mapping. Furthermore, suppose that T is a cyclic (α, β)-admissible mapping and suppose that condition (a_1) holds. If in addition also one of conditions (a_2) and (a_3) is satisfied then T has a fixed point.*

Theorem 1.8. *([2, Theorem 3.3]) If in addition to the hypothesis of Theorem 1.7 we assume that $\alpha(u) \geq 1$ and $\beta(u) \geq 1$ whenever $Tu = u$ then T has a unique fixed point.*

Theorem 1.9. *([2, Theorem 3.6]) Let A and B be two nonempty closed subsets of a b-complete b-metric space (X, d) such that $A \cap B \neq \emptyset$. Let $T : A \cup B \to A \cup B$ be a cyclic mapping. If T is a generalized TAC-cyclic contractive mapping then T has a unique fixed point in $A \cap B$.*

We recall for convenience of reader some immediate consequences of these results.

Corollary 1.10. *([2, Corollary 4.1]) Let (X, d) be a b-complete b-metric space with coefficient $s \geq 1$ and $T : X \to X$. If there exist $\psi \in \Psi$, $\phi \in \Phi$ and $f \in \mathcal{F}$ such that for all $x, y \in X$ the following condition is verified*

$$\psi\left(s^3 d(Tx, Ty)\right) \leq f\left(\psi(M_s(x, y)), \phi(M_s(x, y))\right),$$

then T has a unique fixed point.

Corollary 1.11. *([2, Corollary 4.2]) Let (X,d) be a complete metric space and let $T : X \to X$. Assume there exist $\alpha, \beta : X \to [0,+\infty)$, $\psi \in \Psi$, $\phi \in \Phi$ and $f \in \mathcal{F}$ such that for each x, y in X for which one has $\alpha(x)\beta(y) \geq 1$ the following condition is satisfied*

$$\psi(d(Tx,Ty)) \leq f(\psi(M(x,y)), \phi(M(x,y))),$$

where $M(x,y) = \left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{2}\right\}$. Furthermore, we suppose that T is a cyclic (α,β)-admissible mapping and suppose that condition (a_1) holds. If in addition also one of conditions (a_2) and (a_3) is satisfied then T has a fixed point.

Corollary 1.12. *([2, Corollary 4.3]) Let (X,d) be a complete metric space and let $T : X \to X$. If there exist $\psi \in \Psi$, $\phi \in \Phi$ and $f \in \mathcal{F}$ such that*

$$\psi(d(Tx,Ty)) \leq f(\psi(M(x,y)), \phi(M(x,y)))$$

for all $x, y \in X$, where $M(x,y)$ is defined as in Corollary 1.11, then T has a unique fixed point.

Corollary 1.13. *([2, Corollary 4.4]) Let (X,d) be a b-complete b-metric space with coefficient $s \geq 1$ and let $T : X \to X$. Suppose there exist $\alpha, \beta : X \to [0,+\infty)$, $\psi \in \Psi$ and $\phi \in \Phi$ such that for each x, y in X for which one has $\alpha(x)\beta(y) \geq 1$ the following condition is satisfied*

$$\psi\left(s^3 d(Tx,Ty)\right) \leq \psi(M_s(x,y)) - \phi(M_s(x,y)).$$

Furthermore, suppose that T is a cyclic (α,β)-admissible mapping and suppose that condition (a_1) holds. If in addition also one of conditions (a_2) and (a_3) is satisfied then T has a fixed point.

Corollary 1.14. *([2, Corollary 4.6]) Let A and B be two nonempty closed subsets of a b-complete b-metric space (X,d) with coefficient $s \geq 1$ such that $A \cap B \neq \emptyset$ and let $T : A \cup B \to A \cup B$ be a cyclic mapping. If there exist $\psi \in \Psi$ and $\phi \in \Phi$ such that*

$$\psi\left(s^3 d(Tx,Ty)\right) \leq \psi(M_s(x,y)) - \phi(M_s(x,y)),$$

for all $x \in A$ and $y \in B$ then T has a unique fixed point in $A \cap B$.

In this paper we complement and improve some among of the previous results established by Babu and Dula in the framework of b-metric spaces.

2. Main results

In this section we formulate and prove our main results. The first theorem is given for b-metric spaces with coefficient $s > 1$ and it is a generalization of Theorem 1.7 (see [2], Theorem 3.2). Here, in order to prove our results we use Lemma 2.2 of [14]. In this way, we get slender and nicer proofs than ones in existing literature.

THEOREM 2.1. *Let (X, d) be a b-complete b-metric space with coefficient $s > 1$ and let $T : X \to X$. Assume there exist two mappings $\alpha, \beta : X \to [0, +\infty)$ such that for each $x, y \in X$ for which one has $\alpha(x)\beta(y) \geq 1$ the following condition is satisfied*

$$d(Tx, Ty) \leq \frac{1}{s^3} M_s(x, y) \tag{1}$$

where $M_s(x, y) = \max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x,Ty)+d(y,Tx)}{2s}\right\}$. Furthermore, we suppose that T is a cyclic (α, β)-admissible mapping and we suppose that following condition is verified:

(c_1) there exists $x_0 \in X$ such that $\alpha(x_0) \geq 1$ and $\beta(x_0) \geq 1$.

If in addition also one of the following conditions is satisfied:

(c_2) T is b-continuous,

(c_3) if $\{x_n\}$ is a sequence in X such that $x_n \to x$ and $\beta(x_n) \geq 1$ for all n one has also $\beta(x) \geq 1$,

then T has a fixed point.

Proof. Let x_0 be an element of X that verifies condition (c_1) and let $\{x_n\}$ be the iterative sequence defined by $x_{n+1} = Tx_n$ for $n = 0, 1, 2, \ldots$

We stress that if $x_k = x_{k+1}$ for some $k \in \{0\} \cup \mathbb{N}$ then x_k is a fixed point of T and so we have the claim. Hence, we assume that $x_{n+1} \neq x_n$ for all $n \in \{0\} \cup \mathbb{N}$. Following the proof of [2, Theorem 3.2.], we easily get that $\alpha(x_n) \geq 1$ and $\beta(x_n) \geq 1$ for all $n \in \{0\} \cup \mathbb{N}$. Now by using this, we prove that

$$d(x_{n+1}, x_{n+2}) \leq \lambda\, d(x_n, x_{n+1}) \text{ for all } n \in \{0\} \cup \mathbb{N} \tag{2}$$

where $\lambda \in [0, \frac{1}{s^3}] \subseteq [0, 1)$. Taking into account that $\alpha(x_n)\beta(x_{n+1}) \geq 1$ for all $n \in \{0\} \cup \mathbb{N}$, by using (1), we get

$$d(Tx_n, Tx_{n+1}) \leq \frac{1}{s^3} M_s(x_n, x_{n+1}) \tag{3}$$

where

$$\begin{aligned}
&M_s(x_n, x_{n+1})\\
&= \max\left\{d(x_n, x_{n+1}), d(x_n, Tx_n), d(x_{n+1}, Tx_{n+1}), \frac{1}{2s}\left(d(x_n, Tx_{n+1}) + d(x_{n+1}, Tx_n)\right)\right\}\\
&= \max\left\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2}), \frac{1}{2s}\left(d(x_n, x_{n+2}) + d(x_{n+1}, x_{n+1})\right)\right\}\\
&= \max\left\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2}), \frac{1}{2s} d(x_n, x_{n+2})\right\}\\
&\leq \max\left\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2}), \frac{1}{2}\left(d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2})\right)\right\}\\
&\leq \max\left\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2})\right\}
\end{aligned}$$

and then $M_s(x_n, x_{n+1}) = \max\{d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2})\}$.

We notice that if $d(x_n, x_{n+1}) < d(x_{n+1}, x_{n+2})$ for some $n \in \{0\} \cup \mathbb{N}$ then, by using (3), we have

$$d(x_{n+1}, x_{n+2}) = d(Tx_n, Tx_{n+1}) \leq \frac{1}{s^3} M_s((x_n, x_{n+1})) = \frac{1}{s^3} d(x_{n+1}, x_{n+2}).$$

Clearly, this is a contradiction since $s^3 > 1$. Hence, we conclude that $d(x_{n+1}, x_{n+2}) \leq d(x_n, x_{n+1})$ for all $n \in \{0\} \cup \mathbb{N}$.

Furthermore, by (3) we easily deduce that (2) is also true. Taking into account that $\frac{1}{s^3} \in [0, 1)$, by [14, Lemma 2.2.], we can affirm that $\{x_n\}$ is a b-Cauchy sequence in (X, d). We notice that (X, d) is b-complete and thus there exists $u \in X$ such that $\lim_{n\to\infty} x_n = u$.

Now, we observe that if T is b-continuous (i.e. if (c_2) holds) this assures that $\lim_{n\to\infty} Tx_n = Tu$ and, hence, we get that $Tu = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} x_{n+1} = u$. So, u is a fixed point of T in X. Therefore, we assume that (c_3) is verified. We notice that, since $\beta(x_n) \geq 1$ for all n, condition (c_3) assures that $\beta(u) \geq 1$. We suppose that $Tu \neq u$. By using the triangle inequality, we have that $\frac{1}{s} d(u, Tu) \leq d(u, x_{n+1}) + d(Tx_n, Tu) \leq d(u, x_{n+1}) + \frac{1}{s^3} M_s(x_n, u)$, where

$$\begin{aligned} M_s(x_n, u) &= \max\Big\{d(x_n, u), d(x_n, Tx_n), d(u, Tu), \frac{1}{2s}(d(x_n, Tu) + d(Tx_n, u))\Big\} \\ &\leq \max\Big\{d(x_n, u), d(x_n, Tx_n), d(u, Tu), \frac{1}{2}(d(x_n, u) + d(u, Tu)) + \frac{1}{2s} d(Tx_n, u)\Big\} \\ &= \max\Big\{d(x_n, u), d(x_n, x_{n+1}), d(u, Tu), \frac{1}{2}(d(x_n, u) + d(u, Tu)) + \frac{1}{2s} d(x_{n+1}, u)\Big\} \\ &\to \max\Big\{0, 0, d(u, Tu), \frac{1}{2}(0 + d(u, Tu)) + \frac{1}{2s} \cdot 0\Big\} = d(u, Tu). \end{aligned}$$

Hence, we deduce that $\frac{1}{s} d(u, Tu) \leq \frac{1}{s^3} d(u, Tu)$. Clearly, this is a contradiction since $s > 1$. So, one necessarily has $Tu = u$ and this prove the claim. □

Remark 2.2. We stress that the proof of Theorem 2.1 shows that mappings ψ, ϕ and f in Theorem 1.7 (see [2, Theorem 3.2]) are not necessary.

The next result improves Theorem 1.8 (see [2, Theorem 3.3]) for $s > 1$.

Theorem 2.3. *Let (X, d) be a b-complete b-metric space with coefficient $s > 1$ and let $T : X \to X$, $\alpha, \beta : X \to [0, +\infty)$ be mappings as in Theorem 2.1. Assume that T is a cyclic (α, β)-admissible mapping and condition (c_1) is verified. Further, we assume that $\alpha(u) \geq 1$ and $\beta(u) \geq 1$ whenever $Tu = u$. If in addition one among of conditions (c_2) and (c_3) is verified then T has a unique fixed point.*

Proof. Since Theorem 2.1 assures that T has a fixed point, we must only prove that such a point is unique. We suppose by way of contradiction that u_1 and u_2 are two distinct fixed points of T. Taking into account that by hypothesis $\alpha(u_1) \geq 1$ and $\beta(u_2) \geq 1$, we get by (1) that

$$d(u_1, u_2) = d(Tu_1, Tu_2) \leq \frac{1}{s^3} M_s(u_1, u_2) \tag{4}$$

where

$$M_s(u_1,u_2) = \max\Big\{d(u_1,u_2), d(u_1,Tu_1), d(u_2,Tu_2), \frac{1}{2s}\left(d(u_1,Tu_2)+d(u_2,Tu_1)\right)\Big\}$$
$$= \max\Big\{d(u_1,u_2), 0, 0, \frac{1}{s}\, d(u_1,u_2)\Big\}.$$

Since $s > 1$ we have that $\max\{d(u_1,u_2), 0, 0, \frac{1}{s}\, d(u_1,u_2)\} = d(u_1,u_2)$.

Hence, by using (4) we deduce that $d(u_1,u_2) \leq \frac{1}{s^3}\, d(u_1,u_2)$. Clearly, this is a contradiction and so the claim is proved. □

Now, we notice that Definition 1.6 can be generalized as follows:

Definition 2.4. Let (X, d) be a b-metric space with coefficient $s > 1$ and let A, B be two closed subsets of X such that $X = A \cup B$ and $A \cap B \neq \emptyset$. We call a cyclic mapping $T : X \to X$ a generalized TAC-cyclic contractive mapping if for each $x \in A$ and $y \in B$ the following condition is satisfied

$$d(Tx, Ty) \leq \frac{1}{s^3} M_s(x, y), \tag{5}$$

where $M_s(x,y) = \max\big\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{2s}\big\}$.

By using this generalization of the notion of TAC-contractive mapping, we have the following result which improves Theorem 1.9 (see [2, Theorem 3.6]).

Theorem 2.5. *Let A and B be two nonempty closed subsets of a b-complete b-metric space (X, d) such that $X = A \cup B$ and $A \cap B \neq \emptyset$. Let $T : X \to X$ be a cyclic mapping. If T is a generalized TAC-cyclic contractive mapping (in the sense of Definition 2.4) then T has a unique fixed point in $A \cap B$.*

Proof. Let us denote by $\alpha : A \cup B \to [0, +\infty)$ and $\beta : A \cup B \to [0, +\infty)$ the mappings defined by:

$$\alpha(x) = 1 \text{ if } x \in A \text{ and } \alpha(x) = 0 \text{ if } x \notin A,$$
$$\beta(x) = 1 \text{ if } x \in B \text{ and } \beta(x) = 0 \text{ if } x \notin B.$$

We notice that $\alpha(x)\beta(y) \geq 1$ only if $x \in A$ and $y \in B$. Taking into account that T is a generalized TAC-contractive mapping in the sense of Definition 2.4, we can affirm that (1) is verified. Furthermore, all hypotheses of Theorems 2.1 and 2.3 are satisfied. Hence, we can apply Theorems 2.1 and 2.3 and so we obtain the claim. □

3. Example and conclusion

In this section we give one example in order to support our results.

Example 3.1. Let $X = [0, 1]$ and let $d : X \times X \to [0, +\infty)$ be the mapping defined by $d(x, y) = (x - y)^2$. We notice that (X, d) is a b-complete b-metric space for $s = 2$. Moreover, $X = \left[0, \frac{1}{2}\right] \cup \left[\frac{1}{8}, 1\right]$. We put $A = \left[0, \frac{1}{2}\right]$ and $B = \left[\frac{1}{8}, 1\right]$. Let $T : X \to X$ be the mapping defined by $T(x) = \frac{1-x}{4}$. Since $T(A) = \left[\frac{1}{8}, \frac{1}{4}\right] \subset B$ and

$T(B) = \left[0, \frac{7}{32}\right] \subset A$ the mapping T is cyclic. Furthermore, for each $x \in A$ and $y \in B$ we have

$$2^3 d(Tx, Ty) = 2^3 \left(\frac{1-x}{4} - \frac{1-y}{4}\right)^2 = \frac{1}{2}(x-y)^2 \le (x-y)^2 = d(x,y) \le M_s(x,y).$$

Hence, we can apply Theorem 2.5 and so we deduce that T has a unique fixed point in $A \cap B$. More precisely, the unique fixed point of T in $A \cap B$ is $x = \frac{1}{5}$.

Remark 3.2. Let (X,d) be the b-complete b-metric space of Example 3.1. Moreover, let $A = \left[0, \frac{1}{2}\right]$, $B = \left[\frac{1}{8}, 1\right]$ and let $T : X \to X$ be as in Example 3.1. Let us denote by $\psi, \phi : [0, +\infty) \to [0, +\infty)$ and $f : [0, +\infty)^2 \to \mathbb{R}$ the mappings defined by $\psi(t) = t$, $\phi(t) = \frac{11}{12}t$, $f(s,t) = s - t$ for each $t, s \in [0, +\infty)$. Clearly, ψ, ϕ and f belong to the sets Ψ, Φ, $\mathcal{C}$, respectively.

We stress that Theorem 1.9 (see [2, Theorem 3.6]) cannot be applied in order to prove that T has a unique fixed point in $A \cap B$ with respect to such a choice of functions ψ, ϕ and f. Indeed, we have $2^3 d(Tx, Ty) \le M_s(x,y) - \frac{11}{12} M_s(x,y) = \frac{1}{12} M_s(x,y)$, or equivalently

$$6|x-y|^2 \le \max\left\{(x-y)^2, \frac{(5x-1)^2}{16}, \frac{(5y-1)^2}{16}, \frac{(4x+y-1)^2 + (4y+x-1)^2}{64}\right\}$$

for each $x \in A$, and $y \in B$. Here, choosing $x = 0 \in A$ and $y = 1 \in B$, we get

$$6|0-1|^2 \le \max\left\{(0-1)^2, \frac{(5\cdot 0-1)^2}{16}, \frac{(5\cdot 1-1)^2}{16}, \frac{(4\cdot 0+1-1)^2 + (4\cdot 1+0-1)^2}{64}\right\},$$

i.e. $6 \le \max\left\{1, \frac{1}{16}, 1, \frac{9}{64}\right\} = 1$. Clearly, this is not true.

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