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ORE TYPE CONDITION AND HAMILTONIAN GRAPHS

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Abstract. In 1960, Ore proved that if G is a graph of order $n \geq 3$ such that $d(x)+d(y) \geq n$ for each pair of nonadjacent vertices x, y in G, then G is Hamiltonian. In 1985, Ainouche and Christofides proved that if G is a 2-connected graph of order $n \geq 3$ such that $d(x)+d(y) \geq n-1$ for each pair of nonadjacent vertices x, y in G, then G is Hamiltonian or G belongs to two classes of exceptional graphs. In this paper, we prove that if G is a connected graph of order $n \geq 3$ such that $d(x)+d(y) \geq n-2$ for each pair of nonadjacent vertices x, y in G, then G is Hamiltonian or G belongs to one of several classes of well-structured graphs.

1. Introduction

We consider only finite undirected graphs without loops or multiple edges. For a graph G, let $V(G)$ be the vertex set of G and $E(G)$ the edge set of G. Let K_n denote the complete graph of order n and K_n^- the empty graph of order n. For two vertices u and v, let $d(u,v)$ be the length of a shortest path between vertices u and v in G, that is, $d(u,v)$ is the distance between u and v. We denote by $d(x)$ the degree of vertex x in G and the minimum degree of a graph G is denoted by $\delta(G)$ and the independent number of G is denoted by $\alpha(G)$. For a subgraph H of a graph G and a subset S of $V(G)$, let $N_H(S)$ be the set of vertices in H that are adjacent to some vertex in S, the cardinality of $N_H(S)$ is denoted by $d_H(S)$. In particular, if $H = G$ and $S = \{u\}$, then $N_H(S) = N_G(u)$, which is the neighborhood of u in G. Furthermore, let $G - H$ and $G[S]$ denote the subgraphs of G induced by $V(G) - V(H)$ and S, respectively. For each integer $m \geq 3$, let $C_m = x_1x_2\cdots x_mx_1$ denote a cycle of order m and define

$$N^+_{C_m}(u) = \{x_{i+1} : x_i \in N_{C_m}(u)\}, \quad N^-_{C_m}(u) = \{x_{i-1} : x_i \in N_{C_m}(u)\},$$

$N^{\pm}_{C_m}(u) = N^+_{C_m}(u) \cup N^-_{C_m}(u)$, where subscripts are taken by modulo m.

If no ambiguity can arise we sometimes write $N(u)$ instead of $N_G(u)$, V instead of $V(G)$, etc. We refer to the book [2] for graph theory notation and terminology not described in this paper.

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If a graph G has a Hamiltonian cycle (a cycle containing every vertex of G), then G is called Hamiltonian.

In 1952, Dirac established the well-known degree type condition for Hamiltonian graphs.

THEOREM 1.1. [3] *If the minimum degree of graph G of order n is at least $n/2$, then G is Hamiltonian.*

In 1960, Ore obtained the following Ore type condition:

THEOREM 1.2. [4] *If G is a graph of order $n \geq 3$ such that $d(x) + d(y) \geq n$ for each pair of nonadjacent vertices x, y in G, then G is Hamiltonian.*

In 1985, Ainouche and Christofides proved the following result.

THEOREM 1.3. [1] *If G is a connected graph of order $n \geq 3$ such that $d(x) + d(y) \geq n-1$ for each pair of nonadjacent vertices x, y in G, then G is Hamiltonian or $G \in \{G_1 \vee (K_h \cup K_t), G_{(n-1)/2} \vee K^-_{(n+1)/2}\}$.*

G_h denotes all graphs of order h, h is a positive integer. For graphs A and B the join operator $A \vee B$ of A and B is the graph constructed from A and B by adding all edges joining the vertices of A and the vertices of B. The union operator $A \cup B$ of A and B is the graph of $V(A \cup B) = V(A) \cup V(B)$ and $E(A \cup B) = E(A) \cup E(B)$.

Recently, in [5], [6], some generalized Fan type conditions for Hamiltonian graphs were introduced as follows.

THEOREM 1.4. *If G is a k-connected graph of order n, and if $max\{d(v) : v \in S\} \geq n/2$ for every independent set S of G with $|S| = k$ which has two distinct vertices $x, y \in S$ satisfying $1 \leq |N(x) \cap N(y)| \leq \alpha(G) - 1$, then G is Hamiltonian.*

In this paper, we present the following two results, which improve the above results.

THEOREM 1.5. *If G is a connected graph of order $n \geq 3$ such that $d(x) + d(y) \geq n-2$ for each pair of nonadjacent vertices x, y in G, then G is Hamiltonian or $G \in \{(G_{(n-1)/2} \vee K^-_{(n+1)/2}) - e, G_{(n-1)/2} \vee K^-_{(n+1)/2}, G_{(n-2)/2} \vee (K^-_{(n-2)/2} \cup K_2), G_{(n-2)/2} \vee K^-_{(n+2)/2}, G_2 \vee 3K_2, G_2 \vee (2K_2 \cup K_1), K_1 : C'_6, K_h : w : K'_t, K_{1,3}\}$.*

$3K_2 = K_2 \cup K_2 \cup K_2$. K'_t is the graph obtained from complete graph K_t by removing a matching of size $k \leq t/2$, $(G_{(n-1)/2} \vee K^-_{(n+1)/2}) - e$ is the graphs obtained from graph $G_{(n-1)/2} \vee K^-_{(n+1)/2}$ by removing an edge connected some vertex of $G_{(n-1)/2}$ and some vertex of $K^-_{(n+1)/2}$, graph $K_{1,3}$ is a claw. The two graphs $K_1 : C'_6$ and $K_h : w : K'_t$ can be found in the proofs of Subcase 1.2 and Subcase 2.2 of Theorem 1.5, respectively.

Since Hamiltonian graph is 2-connected, by Theorem 1.5, we have

Corollary 1.6. *If G is a 2-connected graph of order $n \geq 9$ such that $d(x) + d(y) \geq n-2$ for each pair of nonadjacent vertices x, y in G, then G is Hamiltonian or $G \in \{(G_{(n-1)/2} \vee K^-_{(n+1)/2}) - e, (G_{(n-1)/2} \vee K^-_{(n+1)/2}), G_{(n-2)/2} \vee (K^-_{(n-2)/2} \cup K_2), G_{(n-2)/2} \vee K^-_{(n+2)/2}\}$.*

2. The proof of main result

In order to prove Theorem 1.5, we need the following lemma.

Lemma 2.1. *Let G be a 2-connected graph of order $n \geq 3$ such that $d(x) + d(y) \geq n-2$ for each pair of nonadjacent vertices x, y in G. If G is not Hamiltonian and $C_m = x_1x_2 \cdots x_m x_1$ is a longest cycle of G and H is a component of $G - C_m$ with $|V(H)| = |\{u\}| = 1$, than $(n-2)/2 \leq d(u) \leq (n-1)/2$ or $G \in \{G_2 \vee (2K_2 \cup K_1), K_1 : C'_6\}$.*

Proof. Since G is 2-connected, let $x_{i+1}, x_{j+1} \in N^+_{C_m}(u)$. We denote the path $x_{i+1}x_{i+2} \cdots x_j \setminus \{x_j\}$ on C_m by P_1 and the path $x_{j+1}x_{j+2} \cdots x_i$ by P_2. Since C_m is a longest cycle of G, so we have the following,

(i) Each of $N^+_{P_1}(x_{j+1})$ is not adjacent to x_{i+1}(Otherwise, if $x_k \in N^+_{P_1}(x_{j+1})$ is adjacent to x_{i+1}. Let $P(H)$ be a path in H which two end-vertices adjacent to x_i, x_j, respectively, then cycle $x_iP(H)x_jx_{j-1} \cdots x_kx_{i+1}x_{i+2} \cdots x_{k-1}x_{j+1}x_{j+2} \cdots x_i$ is longer than C_m, a contradiction).

(ii) Each of $N^-_{P_2}(x_{j+1})$ is not adjacent to x_{i+1}(Otherwise, if $x_k \in N^-_{P_2}(x_{j+1})$ is adjacent to x_{i+1}. Let $P(H)$ be a path in H which two end-vertices adjacent to x_i, x_j, respectively, then cycle $x_iP(H)x_jx_{j-1} \cdots x_{i+1}x_kx_{k-1} \cdots x_{j+1}x_{k+1}x_{k+2} \cdots x_i$ is longer than C_m, a contradiction). Since $x_j \notin V(P_1)$, so we can see that $N^+_{P_1}(x_{j+1}) \cap N^-_{P_2}(x_{j+1}) = \phi$, and clearly $|N^+_{P_1}(x_{j+1}) \cup N^-_{P_2}(x_{j+1}) \cup \{x_{i+1}\}| \geq |N_{C_m}(x_{j+1})|$.

By (i) and (ii), each of $N^+_{P_1}(x_{j+1}) \cup N^-_{P_2}(x_{j+1}) \cup \{x_{i+1}\}$ is not adjacent to x_{i+1}. Hence we can check $|N_{C_m}(x_{i+1})| \leq |V(C_m)| - |N^+_{P_1}(x_{j+1}) \cup N^-_{P_2}(x_{j+1}) \cup \{x_{i+1}\}| \leq |V(C_m)| - |N_{C_m}(x_{j+1})|$, this implies

$$|N_{C_m}(x_{i+1})| + |N_{C_m}(x_{j+1})| \leq |V(C_m)|. \qquad \text{(i)}$$

Also, both x_{i+1}, x_{j+1} do not have any common neighbor in $G - C_m - H$ and both x_{i+1}, x_{j+1} are not adjacent to any vertex of H. Hence we also have

$$|N_{G-C_m}(x_{i+1})| + |N_{G-C_m}(x_{j+1})| \leq |V(G - C_m - H)|. \qquad \text{(ii)}$$

Combining inequalities (i) and (ii), we have

$$|N(x_{i+1})| + |N(x_{j+1})| \leq |V(C_m)| + |V(G - C_m - H)| \leq n - |V(H)| = n - 1. \qquad \text{(iii)}$$

Then, we claim $d(u) \geq (n-3)/2$. Otherwise, if $d(u) < (n-3)/2$, by the assumption of Lemma that $d(u) + d(x_{i+1}) \geq n-2$ and $d(u) + d(x_{j+1}) \geq n-2$, $d(x_{i+1}) > (n-1)/2$ and $d(x_{j+1}) > (n-1)/2$, so $d(x_{i+1}) + d(x_{j+1}) > n-1$, this contradicts inequality (iii).

Thus, $d(u) \geq (n-3)/2$ holds. Then consider two cases.

When n is even. Since $d(u) \geq (n-3)/2$ and $d(u)$ is integer, so $d(u) \geq (n-2)/2$.

When n is odd. Since $d(u) = (n-3)/2$ and $d(u) \geq 2$, so $(n-3)/2 \geq 2$, this implies $n \geq 7$. (i). When $n \geq 9$, by $d(u) = (n-3)/2$, clearly there exist two vertices x_i, x_{i+2} of C_m that are adjacent to u, then since C_m is a longest cycle, so none of $N^{\pm}_{C_m}(u)$ are adjacent to x_{i+1}. so at most $m - |N^{\pm}_{C_m}(u)|$ vertices are adjacent to x_{i+1}, and clearly $|N^{\pm}_{C_m}(u)| \geq N_{C_m}(u)| + 2$, so we easily check $d(u) + d(x_{i+1}) \leq m - |N^{\pm}_{C_m}(u)| - |N_{C_m}(u)| < n-2$, a contradiction. (ii). When $n = 7, 8$. Since n is odd, so $n \neq 8$, and we only consider $n = 7$. In this case, $d(u) = (n-3)/2 = 2$ and $m = 6$. Let $C_m = x_i x_{i+1} x_{i+2} x_{i+3} x_{i+4} x_{i+5} x_i$. $(ii-1)$. When $d(u) = \{x_i, x_{i+2}\}$. By assumption of Lemma that $d(u) + d(x_k) \geq n-2$ for $k = i+1, i+3, i+4, i+5$ on $C_m = x_i x_{i+1} x_{i+2} x_{i+3} x_{i+4} x_{i+5} x_i$, so $d(x_k) \geq 3$. Since C_m is a longest cycle, $x_{i+1}x_{i+4}, x_{i+3}x_i, x_{i+5}x_{i+2} \in E(G)$. We denote the graph by $K_1 : C'_6$, where $V(K_1 : C'_6) = V(K_1) \cup V(C_6,)$, $E(K_1 : C'_6) = \{ux_i, ux_{i+2}, x_{i+1}x_{i+4}, x_{i+3}x_i, x_{i+5}x_{i+2}\} \cup E(C_6)$. $(ii-2)$. When $d(u) = \{x_i, x_{i+3}\}$. Clearly, to satisfy the assumption of Lemma, each vertex of $G - \{x_i, x_{i+3}\}$ must be adjacent to x_i, x_{i+3}, this implies $G \in G_2 \vee (2K_2 \cup K_1)$, where $V(G_2) = \{x_i, x_{i+3}\}$, $V(K_1) = (u)$, $2K_2 = G[\{x_{i+1}x_{i+2}\}] \cup G[\{x_{i+4}x_{i+5}\}]$.

Thus, this proves that $d(u) \geq (n-2)/2$ or $G \in \{G_2 \vee (2K_2 \cup K_1), K_1 : C'_6\}$.

On the other hand, since C_m is a longest cycle of G, so u is not adjacent to consecutive two vertices on C_m. Hence it can be checked that $d(u) \leq (n-1)/2$.

Therefore, this completes the proof of Lemma. ∎

Proof of Theorem 1.5. Assume that G is not Hamiltonian with G satisfying the assumption of Theorem 5. Let $C_m = x_1 x_2 \cdots x_m x_1$ be a longest cycle of G and H be a component of $G - C_m$. Consider the following cases.

Case 1. The connectivity of G is at least 2.

In this case, since the connectivity of G is at least 2, so there must exist $u, v \in V(H)$ and $x_{i+1}, x_{j+1} \in V(C_m)$ such that $x_{i+1} \in N^{+}_{C_m}(u)$, $x_{j+1} \in N^{+}_{C_m}(v)$(if $|V(H)| = 1$, then $u = v$). We claim $d(x_{i+1}) + d(x_{j+1}) \leq n - |V(H)|$. Otherwise, if $d(x_{i+1}) + d(x_{j+1}) > n - |V(H)|$, then we denote the path $x_{i+1} x_{i+2} \cdots x_j \setminus \{x_j\}$ on C_m by P_1 and the path $x_{j+1} x_{j+2} \cdots x_i$ by P_2. Since C_m is a longest cycle of G, so we have the following, (i). Each of $N^{+}_{P_1}(x_{j+1})$ is not adjacent to x_{i+1}(Otherwise, if $x_k \in N^{+}_{P_1}(x_{j+1})$ is adjacent to x_{i+1}. Let $P(H)$ be a path in H which two end-vertices adjacent to x_i, x_j, respectively, then cycle $x_i P(H) x_j x_{j-1} \cdots x_k x_{i+1} x_{i+2} \cdots x_{k-1} x_{j+1} x_{j+2} \cdots x_i$ is longer than C_m, a contradiction). (ii). Each of $N^{-}_{P_2}(x_{j+1})$ is not adjacent to x_{i+1}(Otherwise, if $x_k \in N^{-}_{P_2}(x_{j+1})$ is adjacent to x_{i+1}. Let $P(H)$ be a path in H which two end-vertices adjacent to x_i, x_j, respectively, then cycle $x_i P(H) x_j x_{j-1} \cdots x_{i+1} x_k x_{k-1} \cdots x_{j+1} x_{k+1} x_{k+2} \cdots x_i$ is longer than C_m, a contradiction). Since $x_j \notin V(P_1)$, so we can see that $N^{+}_{P_1}(x_{j+1}) \cap N^{-}_{P_2}(x_{j+1}) = \phi$, and clearly $|N^{+}_{P_1}(x_{j+1}) \cup N^{-}_{P_2}(x_{j+1}) \cup \{x_{i+1}\}| \geq |N_{C_m}(x_{j+1})|$. By (i) and (ii), each of $N^{+}_{P_1}(x_{j+1}) \cup N^{-}_{P_2}(x_{j+1}) \cup \{x_{i+1}\}$ is not adjacent to x_{i+1}. Hence it can be checked that $|N_{C_m}(x_{i+1})| \leq |V(C_m)| - |N^+_{P_1}(x_{j+1}) \cup N^-_{P_2}(x_{j+1}) \cup \{x_{i+1}\}| \leq |V(C_m)| - |N_{C_m}(x_{j+1})|$, this implies

$$|N_{C_m}(x_{i+1})| + |N_{C_m}(x_{j+1})| \leq |V(C_m)|. \quad (1)$$

Also, both x_{i+1}, x_{j+1} do not have any common neighbor in $G - C_m - H$ and both x_{i+1}, x_{j+1} are not adjacent to any vertex of H. Hence we also have

$$|N_{G-C_m}(x_{i+1})| + |N_{G-C_m}(x_{j+1})| \leq |V(G - C_m - H)|. \quad (2)$$

Combining inequalities (1) and (2), we have

$$|N(x_{i+1})| + |N(x_{j+1})| \leq |V(C_m)| + |V(G - C_m - H)| \leq n - |V(H)|. \quad (3)$$

Therefore, the above claim that $d(x_{i+1}) + d(x_{j+1}) \leq n - |V(H)|$ holds.

Then, by the assumption of Theorem that $d(x_{i+1}) + d(x_{j+1}) \geq n-2$, together with above claim, we have $|V(H)| \leq 2$.

Now, we consider the following subcases on $|V(H)| \leq 2$.

Subcase 1.1. When $|V(H)| = 2$.

In this case let $V(H) = \{u, v\}$. Since C_m is a longest cycle of G, so clearly $|\{x_i, x_{i+1}, \cdots x_{j-1}\}| \geq 3$ for each pairs vertices x_i, x_j that $x_i \in N_{C_m}(u), x_j \in N_{C_m}(v)$, thus we can check $d(u) \leq |V(C_m)|/3 + |V(H-u)| \leq (n-2)/3 + 1$. Then by the assumption of Theorem that $d(u) + d(x_{i+1}) \geq n - 2$, we have $d(x_{i+1}) \geq (2n-7)/3$, Similarly, also $d(x_{j+1}) \geq (2n-7)/3$. Hence we have

$$d(x_{i+1}) + d(x_{j+1}) \geq (4n - 14)/3. \quad (4)$$

When $n \geq 9$, clearly, inequality (4) contradicts inequality (3).

When $n \leq 8$, we consider the following two cases. (i).If $n \leq 7$. In this case, since $|V(H)| = 2$, then $m \leq 5$, so there must exist two consecutive vertices x_i, x_{i+1} or two vertices x_i, x_{i+2} on C_m that are adjacent to u, v, respectively. Hence we easily obtain a cycle longer than C_m, a contradiction. (ii). If $n = 8$. Then clearly $m = 6$. By $|\{x_i, x_{i+1}, \cdots x_{j-1}\}| \geq 3$ for each pairs $x_i \in N_{C_m}(u), x_j \in N_{C_m}(v)$. When u is adjacent to vertex x_i on C_m, then since C_m is a longest cycle, so v is at most adjacent to both x_i, x_{i+3}. Again then, clearly u is also at most adjacent to both x_i and x_{i+3}. Since C_m is a longest cycle, so each vertex of $\{x_{i+1}, x_{i+2}\}$ is not adjacent to any of $\{x_{i+4}, x_{i+5}\}$, this implies $G \in G_2 \vee 3K_2$, where $3K_2 = H \cup G[\{x_{i+1}, x_{i+2}\}] \cup G[\{x_{i+4}, x_{i+5}\}]$, $G_2 = G[\{x_i, x_{i+3}\}]$.

Subcase 1.2. When $|V(H)| = 1$.

In this case, let $V(H) = \{u\}$. By Lemma 2.1, $(n-2)/2 \leq d(u) \leq (n-1)/2$ or $G \in \{G_2 \vee (2K_2 \cup K_1), K_1 : C'_6\}$. Thus, we only consider $(n-2)/2 \leq d(u) \leq (n-1)/2$.

Subcase 1.2.1. $d(u) = (n-2)/2$.

When $|V(G - C_m)| = 1$. In this case since G has not Hamiltonian cycle C_n and $d(u) = (n-2)/2$, so we easy to obtain $N(u) = \{x_i, x_{i+3}, x_{i+5}, x_{i+7}, \cdots, x_{i-2}\}$

on C_m, i.e., in $\{x_i, x_{i+3}, x_{i+5}, x_{i+7}, \cdots, x_{i-2}\}$ the first two vertices are x_i, x_{i+3} and all other vertices are $x_{i+5}, x_{i+7}, \cdots, x_{i-2}$.

In this case since G has not Hamiltonian cycle C_n, clearly $\{x_{i+4}, x_{i+6}, \cdots, x_{i-1}, u\}$ is a independent set and each of $\{x_{i+1}, x_{i+2}\}$ is not adjacent to any of $\{x_{i+4}, x_{i+6}, \cdots, x_{i-1}, u\}$, this implies $G \in G_{(n-2)/2} \vee (K^-_{(n-2)/2} \cup K_2)$, where $V(G_{(n-2)/2}) = \{x_i, x_{i+3}, x_{i+5}, x_{i+7}, \cdots, x_{i-1}\}$, $V(K^-_{(n-2)/2}) = \{x_{i+4}, x_{i+6}, \cdots, x_{i-1}, u\}$, $V(K_2) = \{x_{i+1}, x_{i+2}\}$.

When $|V(G - C_m)| = 2$. In this case let $v \in V(G - C_m - u)$. Since G has not Hamiltonian cycle C_n and $d(u) = (n-2)/2$, so we easily obtain $N(u) = \{x_{i+1}, x_{i+3}, \cdots, x_{i-1}\}$, this implies $G \in G_{(n-2)/2} \vee K^-_{(n+2)/2}$, where $V(G_{(n-2)/2}) = \{x_{i+1}, x_{i+3}, \cdots, x_{i-1}\}$, $V(K^-_{(n+2)/2}) = \{x_{i+2}, x_{i+4}, \cdots, x_i, u, v\}$.

Subcase 1.2.2. When $d(u) = (n-1)/2$.

In this case, since C_m is a longest cycle of G, we easily obtain $G \in G_{(n-1)/2} \vee K^-_{(n+1)/2}$ or $G \in (G_{(n-1)/2} \vee K^-_{(n+1)/2}) - e$, where e is an edge connected by some two vertices u and v with u in $G_{(n-1)/2}$ and v in $K^-_{(n+1)2}$.

Case 2. The connectivity of G is 1.

In this case, let w be a cut vertex of G and let H', H'' be two components of $G - w$.

Subcase 2.1. $|V(G - H' - H'')| > |\{w\}|$.

In this case, let H''' be a component of $(G - w) - H' - H''$, and let $x \in V(H')$, $y \in V(H'')$ and $z \in V(H''')$. Without loss of generality, assume $|V(H')| = max\{|V(H')|, |V(H'')|, |V(H''')|\}$, then clearly $|V(H'')| + |V(H''')| \leq 2(n-1)/3$, so we can check that $d(y) + d(z) \leq 2(n-1)/3 + 2|\{w\}| - |\{y\}| - |\{z\}| = (2n-2)/3$, by $n \geq 7$, so $(2n-2)/3 \leq n - 3$, a contradiction.

Subcase 2.2. $|V(G - H' - H'')| = |\{w\}| = 1$.

When $n \geq 7$. In this case, without loss of generality, assume $|V(H')| \geq |V(H'')|$, then H'' is a complete (Otherwise, if there exist two nonadjacent vertices u, v in H'', we can check $d(u) + d(v) \leq n - 3$, a contradiction). Let $u \in V(H'')$, by $d(x) + d(u) \geq n - 2$ for each vertex x in H', x is at most not adjacent to a vertex of $H' \setminus \{x\}$, thus, $G \in H' : w : H'' = K_h : w : K'_t$, where K_h is complete graph of order h and K'_t can be obtained from a complete graph K_t by removing a matching of size $0 \leq k \leq t/2$(i.e., K'_t is the graph by removing some vertex disjoint edges of K_t), with $1 \leq h \leq (n-1)/2$, $t = n - h - 1$. In particular, if $h = (n-1)/2$, then $G \in K_{(n-1)/2} : w : K_{(n-1)/2}$.

When $n = 6$. in this case we easily obtain $G \in K_h : w : K'_t$, where $h + t = 5$.

When $n = 5$, similarly, we have $G \in K_h : w : K'_t$, where $h + t = 4$.

When $n = 4$, we easily obtain $G \in K_h : w : K'_t$, where $h + t = 3$ or $G - w$ consists of three components, so in this case G is a claw-free graph $K_{1,3}$.

When $n = 3$, clearly, $G \in K_h : w : K'_t = K_{1,2}$.

Therefore, this completes the proof of Theorem. ■

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