

COMMON FIXED POINT RESULT FOR TWO SELF-MAPS IN G -METRIC SPACES

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Abstract. In this paper, we prove a fixed point result for two maps in a generalized metric space (X, G) . Also, we prove the uniqueness of such fixed point. This result generalizes some well known results in the literature due to M. Abbas and B.E. Rhoades [*Common fixed point results for noncommuting mapping without continuity in generalized metric spaces*, Appl. Math. Computation **215** (2009), 262–269] and W. Shatanawi [*Fixed point theory for contractive mappings satisfying Φ -maps in G -metric spaces*, Fixed Point Theory Appl. 2010 (2010), Article ID 181650, 9 pages].

1. Introduction

The study of common fixed points of mappings satisfying certain contractive conditions has been at the center of strong research activity, being the area of the fixed point theory has very important application in applied mathematics and sciences. In 1976, Jungck [4] proved a common fixed point theorem for commuting maps, generalizing the Banach contraction principle. This theorem has many applications in mathematics. While Kannan [16] proved the existence of a fixed point for a map that can have a discontinuity in its domain; however, the maps involved were continuous at the fixed point. The notion of weakly commuting maps is introduced by Sessa [15]. Jungck [5] generalized the concept of weak commutativity and showed that weakly commuting maps are compatible but the converse is not true. The concept of R-weakly commuting maps is defined by Pant [14]. Jungck [7] defined a pair of self mappings to be weakly compatible if they commute at their coincidence points. In recent years, several authors have obtained coincidence point results for various classes of mappings on a metric space, utilizing these concepts. For a survey of coincidence point theory we refer to [1, 3, 6, 8, 14, 17, 18]. Mustafa and Sims [11] introduced a new notion of generalized metric space called G -metric spaces. For more results on G -metric spaces, we refer the reader to [1, 2, 10–13, 17–20]. Recently, Abbas and Rhoades [1] obtained some common fixed point theorems for non commuting maps without continuity satisfying different contractive

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conditions in the setting of generalized metric spaces. In this paper, we prove a fixed point theorem for non commuting maps without continuity in generalized metric space. Our result generalizes Theorems 2.3 and 2.4 of [1].

2. Basic concepts

The following definition was introduced by Mustafa and Sims.

DEFINITION 2.1. [11] Let X be a nonempty set and let $G : X \times X \times X \rightarrow \mathbf{R}^+$ be a function satisfying the following properties:

- (G₁) $G(x, y, z) = 0$ iff $x = y = z$,
- (G₂) $0 < G(x, x, y)$, for all $x, y \in X$ with $x \neq y$,
- (G₃) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$,
- (G₄) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \dots$, symmetry in all three variables,
- (G₅) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.

Then the function G is called a generalized metric, or, more specifically, a G -metric on X , and the pair (X, G) is called a G -metric space.

DEFINITION 2.2. [11] Let (X, G) be a G -metric space, and let (x_n) be a sequence of points of X . A point $x \in X$ is said to be the limit of the sequence (x_n) , if $\lim_{n, m \rightarrow +\infty} G(x, x_n, x_m) = 0$, and we say that the sequence (x_n) is G -convergent to x or (x_n) G -converges to x .

Thus, $x_n \rightarrow x$ in a G -metric space (X, G) if for any $\varepsilon > 0$, there exists $k \in \mathbf{N}$ such that $G(x, x_n, x_m) < \varepsilon$ for all $m, n \geq k$.

PROPOSITION 2.1. [11] *Let (X, G) be a G -metric space. Then the following are equivalent:*

1. (x_n) is G -convergent to x .
2. $G(x_n, x_n, x) \rightarrow 0$ as $n \rightarrow +\infty$.
3. $G(x_n, x, x) \rightarrow 0$ as $n \rightarrow +\infty$.
4. $G(x_n, x_m, x) \rightarrow 0$ as $n, m \rightarrow +\infty$.

DEFINITION 2.3. [11] Let (X, G) be a G -metric space. A sequence (x_n) is called G -Cauchy if for every $\varepsilon > 0$, there is $k \in \mathbf{N}$ such that $G(x_n, x_m, x_l) < \varepsilon$, for all $n, m, l \geq k$; that is $G(x_n, x_m, x_l) \rightarrow 0$ as $n, m, l \rightarrow +\infty$

PROPOSITION 2.2. [10] *Let (X, G) be a G -metric space. Then the following are equivalent:*

1. The sequence (x_n) is G -Cauchy.
2. For every $\epsilon > 0$, there is $k \in \mathbf{N}$ such that $G(x_n, x_m, x_m) < \epsilon$, for all $n, m \geq k$.

DEFINITION 2.4. [11] Let (X, G) and (X', G') be G -metric spaces, and let $f : (X, G) \rightarrow (X', G')$ be a function. Then f is said to be G -continuous at a point $a \in X$ if and only if for every $\varepsilon > 0$, there is $\delta > 0$ such that $x, y \in X$ and

$G(a, x, y) < \delta$ implies $G'(f(a), f(x), f(y)) < \varepsilon$. A function f is G -continuous at X if and only if it is G -continuous at all $a \in X$.

PROPOSITION 2.3. [11] *Let (X, G) be a G -metric space. Then the function $G(x, y, z)$ is jointly continuous in all three of its variables.*

The following are examples of G -metric spaces.

EXAMPLE 2.1. [11] Let (\mathbf{R}, d) be the usual metric space. Define G_s by

$$G_s(x, y, z) = d(x, y) + d(y, z) + d(x, z)$$

for all $x, y, z \in \mathbf{R}$. Then it is clear that (\mathbf{R}, G_s) is a G -metric space.

EXAMPLE 2.2. [11] Let $X = \{a, b\}$. Define G on $X \times X \times X$ by

$$G(a, a, a) = G(b, b, b) = 0, \quad G(a, a, b) = 1, \quad G(a, b, b) = 2$$

and extend G to $X \times X \times X$ by using the symmetry in the variables. Then it is clear that (X, G) is a G -metric space.

DEFINITION 2.5. [11] A G -metric space (X, G) is called G -complete if every G -Cauchy sequence in (X, G) is G -convergent in (X, G) .

DEFINITION 2.6. Let f and g be self maps of a set X . If $w = fx = gx$ for some x in X , then x is called a coincidence point of f and g , and w is called a point of coincidence of f and g .

PROPOSITION 2.4. [1] *Let f and g be weakly compatible self maps of a set X . If f and g have a unique point of coincidence $w = fx = gx$, then w is the unique common fixed point of f and g .*

3. Main results

Following Matkowski [9], let Φ be the set of all functions ϕ such that $\phi : [0, +\infty) \rightarrow [0, +\infty)$ is a nondecreasing function with $\lim_{n \rightarrow +\infty} \phi^n(t) = 0$ for all $t \in (0, +\infty)$. If $\phi \in \Phi$, then ϕ is called a Φ -map. If ϕ is a Φ -map, then it is easy matter to show that:

1. $\phi(t) < t$ for all $t \in (0, +\infty)$.
2. $\phi(0) = 0$.

In the rest of this paper, by ϕ we mean a Φ -map. Now, we introduce and prove our main result.

THEOREM 3.1. *Let X be a G -metric space. Suppose the maps $f, g : X \rightarrow X$ satisfy:*

$$G(fx, fy, fz) \leq \phi(\max\{G(gx, gy, gz), G(gx, fx, fx), G(gy, fy, fy), G(gz, fz, fz)\}) \quad (1)$$

or

$$G(fx, fy, fz) \leq \phi(\max\{G(gx, gy, gz), G(gx, gx, fx), G(gy, gy, fy), G(gz, gz, fz)\}) \quad (2)$$

for all $x, y, z \in X$. If $f(X) \subseteq g(X)$ and $g(X)$ is a G -complete subspace of X , then f and g have a unique point of coincidence in X . Moreover, if f and g are weakly compatible, then f and g have a unique common fixed point.

Proof. Suppose f and g satisfy inequality (1). Let x_0 be an arbitrary point in X . Since $f(X) \subseteq g(X)$, choose $x_1 \in X$ such that $fx_0 = gx_1$. Continuing this process, we produce a sequence (x_n) in X such that $fx_n = gx_{n+1}$ for all $n \in \mathbf{N}$. For $n \in \mathbf{N} \cup \{0\}$, we have

$$\begin{aligned} G(gx_n, gx_{n+1}, gx_{n+1}) &= G(fx_{n-1}, fx_n, fx_n) \\ &\leq \phi(\max\{G(gx_{n-1}, gx_n, gx_n), G(gx_{n-1}, fx_{n-1}, fx_{n-1}), \\ &\quad G(gx_n, fx_n, fx_n), G(gx_n, fx_n, fx_n)\}). \end{aligned}$$

Since $G(gx_n, fx_n, fx_n) = G(gx_n, gx_{n+1}, gx_{n+1})$ and $\phi(G(gx_n, fx_n, fx_n)) < G(gx_n, gx_{n+1}, gx_{n+1})$, we have

$$\begin{aligned} \max\{G(gx_{n-1}, gx_n, gx_n), G(gx_{n-1}, fx_{n-1}, fx_{n-1}), G(gx_n, fx_n, fx_n)\} \\ = G(gx_{n-1}, gx_n, gx_n). \end{aligned}$$

Thus for $n \in \mathbf{N} \cup \{0\}$, we have

$$\begin{aligned} G(gx_n, gx_{n+1}, gx_{n+1}) &\leq \phi(G(gx_{n-1}, gx_n, gx_n)) \\ &\leq \phi^2(G(gx_{n-2}, gx_{n-1}, gx_{n-1})) \\ &\vdots \\ &\leq \phi^n(G(gx_0, gx_1, gx_1)). \end{aligned}$$

Given $\epsilon > 0$. Since $\lim_{n \rightarrow +\infty} \phi^n(G(gx_0, gx_1, gx_1)) = 0$ and $\frac{1}{2}(\epsilon - \phi(\epsilon)) > 0$, there is an integer k_0 such that

$$\phi^n(G(gx_0, gx_1, gx_1)) < \frac{1}{2}(\epsilon - \phi(\epsilon)) \text{ for all } n \geq k_0.$$

Hence

$$G(gx_n, gx_{n+1}, gx_{n+1}) < \frac{1}{2}(\epsilon - \phi(\epsilon)) \text{ for all } n \geq k_0. \quad (3)$$

For $k, n \in \mathbf{N} \cup \{0\}$ with $k > n$, we claim:

$$G(gx_n, gx_k, gx_k) < \epsilon \text{ for all } k \geq n \geq k_0. \quad (4)$$

We prove inequality (4) by induction on k . inequality (4) holds for $k = n + 1$ by using inequality (3) and the fact that $\frac{1}{2}(\epsilon - \phi(\epsilon)) < \epsilon$. Assume inequality (4) holds for $k = m$, that is,

$$G(gx_n, gx_m, gx_m) < \epsilon \text{ for all } m \geq n \geq k_0. \quad (5)$$

For $k = m + 1$, we have

$$G(gx_n, gx_{m+1}, gx_{m+1}) \leq G(gx_n, gx_{n+1}, gx_{n+1}) + G(gx_{n+1}, gx_{m+1}, gx_{m+1})$$

From inequality (1), we have

$$\begin{aligned} G(gx_{n+1}, gx_{m+1}, gx_{m+1}) &= G(fx_n, fx_m, fx_m) \\ &\leq \phi(\max\{G(gx_n, gx_m, gx_m), G(gx_n, fx_n, fx_n), G(gx_m, fx_m, fx_m)\}). \end{aligned}$$

If

$$\begin{aligned} \max\{G(gx_n, gx_m, gx_m), G(gx_n, fx_n, fx_n), G(gx_m, fx_m, fx_m)\} \\ = G(gx_n, gx_m, gx_m), \end{aligned}$$

then

$$G(gx_n, gx_{m+1}, gx_{m+1}) \leq G(gx_n, gx_{n+1}, gx_{n+1}) + \phi(G(gx_n, gx_m, gx_m)).$$

By inequalities (3) and (5), we get

$$G(gx_n, gx_{m+1}, gx_{m+1}) < \frac{1}{2}(\epsilon - \phi(\epsilon)) + \phi(\epsilon) < \epsilon.$$

If

$$\begin{aligned} \max\{G(gx_n, gx_m, gx_m), G(gx_n, fx_n, fx_n), G(gx_m, fx_m, fx_m)\} \\ = G(gx_n, fx_n, fx_n), \end{aligned}$$

then

$$\begin{aligned} G(gx_n, gx_{m+1}, gx_{m+1}) &\leq G(gx_n, gx_{n+1}, gx_{n+1}) + \phi(G(gx_n, fx_n, fx_n)) \\ &\leq 2G(gx_n, gx_{n+1}, gx_{n+1}). \end{aligned}$$

By inequality (3), we get

$$G(gx_n, gx_{m+1}, gx_{m+1}) < \epsilon - \phi(\epsilon) < \epsilon.$$

If

$$\begin{aligned} \max\{G(gx_n, gx_m, gx_m), G(gx_n, fx_n, fx_n), G(gx_m, fx_m, fx_m)\} \\ = G(gx_m, fx_m, fx_m), \end{aligned}$$

then

$$G(gx_n, gx_{m+1}, gx_{m+1}) \leq G(gx_n, gx_{n+1}, gx_{n+1}) + \phi(G(gx_m, fx_m, fx_m)).$$

Since $\phi(G(gx_m, fx_m, fx_m)) < G(gx_m, fx_m, fx_m)$ and $m > n \geq k_0$, then by inequality (3) we have

$$G(gx_n, gx_{m+1}, gx_{m+1}) < \epsilon - \phi(\epsilon) < \epsilon.$$

By induction on k , we conclude that inequality (2) holds for all $k \geq n \geq k_0$. So (gx_n) is a G -Cauchy sequence in $g(X)$. Since $g(X)$ is G -complete, there is a point q in $g(X)$ such that (gx_n) is G -convergent to some q . Choose $p \in X$ such that $gp = q$. We claim $fp = gp$. If not, then for $n \in \mathbf{N} \cup \{0\}$ we have

$$\begin{aligned} G(gx_n, fp, fp) &= G(fx_{n-1}, fp, fp) \\ &\leq \phi(\max\{G(gx_{n-1}, gp, gp), G(gx_{n-1}, fx_{n-1}, fx_{n-1}), G(gp, fp, fp)\}). \end{aligned}$$

If

$$\max\{G(gx_{n-1}, gp, gp), G(gx_{n-1}, fx_{n-1}, fx_{n-1}), G(gp, fp, fp)\} = G(gx_{n-1}, gp, gp),$$

then

$$G(gx_n, fp, fp) \leq \phi(G(gx_{n-1}, gp, gp)) < G(gx_{n-1}, gp, gp).$$

Letting $n \rightarrow +\infty$ and using the fact that G is continuous on its variables, we get that $gp = fp$. If

$$\begin{aligned} \max\{G(gx_{n-1}, gp, gp), G(gx_{n-1}, fx_{n-1}, fx_{n-1}), G(gp, fp, fp)\} \\ = G(gx_{n-1}, fx_{n-1}, fx_{n-1}), \end{aligned}$$

then

$$G(gx_n, fp, fp) \leq \phi(G(gx_{n-1}, fx_{n-1}, fx_{n-1})) = \phi(G(gx_{n-1}, gx_n, gx_n)).$$

Since (gx_n) is G -Cauchy and $\phi(G(gx_{n-1}, gx_n, gx_n)) < G(gx_{n-1}, gx_n, gx_n)$, by letting $n \rightarrow +\infty$, we get $gp = fp$. If

$$\max\{G(gx_{n-1}, gp, gp), G(gx_{n-1}, fx_{n-1}, fx_{n-1}), G(gp, fp, fp)\} = G(gp, fp, fp),$$

then $G(gx_n, fp, fp) \leq \phi(G(gp, fp, fp))$. Letting $n \rightarrow +\infty$ and using the fact that G is continuous on its variables, we get

$$G(gp, fp, fp) \leq \phi(G(gp, fp, fp)).$$

Since $\phi(G(gp, fp, fp)) < G(gp, fp, fp)$, we have $G(gp, fp, fp) < G(gp, fp, fp)$ which is a contradiction. So $gp = fp$. To show that p is unique. Assume that there exists another q in X such that $fq = gq$. If $gp \neq gq$, then we have

$$\begin{aligned} G(gq, gp, gp) &= G(fq, fp, fp) \\ &\leq \phi(\max\{G(gq, gp, gp), G(gq, fq, fq), G(gp, fp, fp)\}). \end{aligned}$$

Since $G(gq, fq, fq) = 0$, $G(gp, fp, fp) = 0$, and $\phi(G(gq, gp, gp)) < G(gq, gp, gp)$, we have $G(gq, gp, gp) < G(gq, gp, gp)$ which is a contradiction. So $gp = gq$. From Proposition 2.4, f and g have a unique common fixed point. The proof using inequality (2) is similar. ■

Theorem 3.1 generalizes Theorems 2.3 and 2.4 [1]:

COROLLARY 3.1. [1] *Let X be a G -metric space. Suppose the maps $f, g : X \rightarrow X$ satisfy:*

$$G(fx, fy, fz) \leq aG(gx, gy, gz) + bG(gx, fx, fx) + cG(gy, fy, fy) + dG(gz, fz, fz)$$

or

$G(fx, fy, fz) \leq aG(gx, gy, gz) + bG(gx, gx, fx) + cG(gy, gy, fy) + dG(gz, gz, fz)$ for all $x, y, z \in X$, where $a + b + c + d < 1$. If $f(X) \subseteq g(X)$ and $g(X)$ is a G -complete subspace of X , then f and g have a unique point of coincidence in X . Moreover, if f and g are weakly compatible, then f and g have a unique common fixed point.

Proof. For $x, y, z \in X$, let

$$M(x, y, z) = \max\{G(gx, gy, gz), G(gx, fx, fx), G(gy, fy, fy), G(gz, fz, fz)\}.$$

Then

$$\begin{aligned} aG(gx, gy, gz) + bG(gx, fx, fx) + cG(gy, fy, fy) + dG(gz, fz, fz) \\ \leq (a + b + c + d)M(x, y, z). \end{aligned}$$

So, if

$$G(fx, fy, fz) \leq aG(gx, gy, gz) + bG(gx, fx, fx) + cG(gy, fy, fy) + dG(gz, fz, fz),$$

then $G(fx, fy, fz) \leq (a + b + c + d)M(x, y, z)$. Define $\phi : [0, +\infty) \rightarrow [0, +\infty)$ by $\phi(t) = (a + b + c + d)t$. Then ϕ is a nondecreasing function. Also, if $a + b + c + d < 1$ then $\lim_{n \rightarrow +\infty} \phi^n(t) = 0$ for all $t > 0$. Hence by Theorem 3.1, we get the result. ■

The above corollary is Theorem 2.3 of [1], which is itself a generalization of a result of [13].

COROLLARY 3.2. [1] *Let X be a G -metric space. Suppose the maps $f, g : X \rightarrow X$ satisfy:*

$$G(fx, fy, fz) \leq k \max\{G(gx, fx, fx), G(gy, fy, fy), G(gz, fz, fz)\}$$

or

$$G(fx, fy, fz) \leq k \max\{G(gx, gx, fx), G(gy, gy, fy), G(gz, gz, fz)\}$$

for all $x, y, z \in X$, where $0 \leq k < 1$. If $f(X) \subseteq g(X)$ and $g(X)$ is a G -complete subspace of X , then f and g have a unique point of coincidence in X . Moreover, if f and g are weakly compatible, then f and g have a unique common fixed point.

Proof. For all $x, y, z \in X$, we let

$$M(x, y, z) = \max\{G(gx, gy, gz), G(gx, fx, fx), G(gy, fy, fy), G(gz, fz, fz)\}.$$

If

$$G(fx, fy, fz) \leq k \max\{G(gx, fx, fx), G(gy, fy, fy), G(gz, fz, fz)\},$$

then $G(fx, fy, fz) \leq kM(x, y, z)$. Define $\phi : [0, +\infty) \rightarrow [0, +\infty)$ by $\phi(t) = kt$. Then it's clear that ϕ is nondecreasing and $\lim_{n \rightarrow +\infty} \phi^n(t) = 0$ for all t . The result follows from Theorem 3.1. ■

The above corollary is Theorem 2.4 of [1], which is itself a generalization of a result of [13].

Theorem 3.1, generalizes Theorem 3.1 of [18].

COROLLARY 3.3. [18] *Let X be a complete G -metric space. Suppose the map $T : X \rightarrow X$ satisfies:*

$$G(T(x), T(y), T(z)) \leq \phi(G(x, y, z))$$

for all $x, y, z \in X$. Then T has a unique fixed point (say u) and T is G -continuous at u .

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