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IMPLICIT APPROXIMATION METHODS FOR COMMON FIXED POINTS OF A FINITE FAMILY OF STRICTLY PSEUDOCONTRACTIVE MAPPINGS IN BANACH SPACES

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Abstract. The purpose of this paper is to present some new implicit approximation methods for finding a common fixed point of a finite family of strictly pseudocontractive mappings in q-uniformly smooth and uniformly convex Banach spaces.

1. Introduction

Let X be a q-uniformly smooth Banach space which is also uniformly convex and its dual space X^* be strictly convex. For the sake of simplicity, the norms of X and X^* are denoted by the symbol $\|\cdot\|$. We write $\langle x, x^* \rangle$ instead of $x^*(x)$ for $x^* \in X^*$ and $x \in X$. Let $\{T_i\}_{i=1}^N$ be a family of strictly pseudocontractive mappings in X with the domain of definition $D(T_i) = X$.

Consider the following problem: find an element

$$x_* \in S := \bigcap_{i=1}^{N} F(T_i), \tag{1.1}$$

where $F(T_i)$ denotes the set of fixed points of the mapping T_i in X. In this paper we assume that $S \neq \emptyset$.

Recall that a mapping T_i in X is called strictly pseudocontractive in the terminology of Browder and Petryshyn [2] if for all $x, y \in D(T_i)$, there exists $\lambda_i > 0$ such that

$$\langle T_i(x) - T_i(y), j(x-y) \rangle \leq \|x-y\|^2 - \lambda_i \|x - y - (T(x) - T(y))\|^2, \tag{1.2}$$

where $j(x)$ denotes the normalized duality mapping of the space X. If I denotes the identity operator in X, then (1.2) can be written in the form

$$\langle (I-T_i)(x) - (I-T_i)(y), j(x-y) \rangle \geq \lambda_i \|(I-T_i)(x) - (I-T_i)(y)\|^2. \tag{1.3}$$

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In the Hilbert space H, (1.2) (and hence (1.3)) is equivalent to the inequality

$$\|T_i(x) - T_i(y)\|^2 \leq \|x - y\|^2 + k_i\|(I - T_i)(x) - (I - T_i)(y)\|^2, \quad k_i = 1 - \lambda_i.$$

Clearly, when $k_i = 0$, T_i is nonexpansive, i.e.,

$$\|T_i(x) - T_i(y)\| \leq \|x - y\|.$$

Wang [11] proved the following result.

THEOREM 1.1. *Let H be a Hilbert space, $T\colon H \to H$ a nonexpansive mapping with $F(T) \neq \emptyset$, and $F\colon H \to H$ an η-strongly monotone and k-Lipschitzian mapping. For any $x_0 \in H$, $\{x_n\}$ is defined by*

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n)T^{\lambda_{n+1}} x_n, \quad n \geq 0,$$

where $\{\alpha_n\}$ and $\{\lambda_n\} \subset [0, 1)$ satisfy the following conditions:
(1) $\alpha \leq \alpha_n \leq \beta$ for some $\alpha, \beta \in (0, 1)$; (2) $\sum_{n=1}^{\infty} \lambda_n < +\infty$; (3) $0 < \mu < 2\eta/k^2$. Then,
(1) $\{x_n\}$ converges weakly to a fixed point of T;
(2) $\{x_n\}$ converges strongly to a fixed point of T if and only if

$$\liminf_{n\to\infty} d(x_n, F(T)) = 0.$$

Zeng and Yao [13] proved the following results.

THEOREM 1.2. *Let H be a Hilbert space, $F\colon H \to H$ be a mapping such that for some constants $k, \eta > 0$, F is k-Lipschitzian and η-strongly monotone. Let $\{T_i\}_{i=1}^N$ be N nonexpansive self-maps of H such that $C = \bigcap_{i=1}^N F(T_i) \neq \emptyset$. Let $\mu \in (0, 2\eta/k^2)$, let $x_0 \in H$, $\{\lambda_n\}_{n=1}^{\infty} \subset [0, 1)$ and $\{\alpha_n\}_{n=1}^{\infty} \subset (0, 1)$ satisfying the conditions: $\sum_{n=1}^{\infty} \lambda_n < \infty$ and $\alpha \leq \alpha_n \leq \beta$, $n \geq 1$, for some $\alpha, \beta \in (0, 1)$. Then the sequence $\{x_n\}$ defined by*

$$\begin{aligned} x_n &= \alpha_{n-1}x_{n-1} + (1 - \alpha_n)T_n^{\lambda_n} x_n \\ &= \alpha_{n-1}x_{n-1} + (1 - \alpha_n)[T_n x_n - \lambda_n \mu F(T_n x_n)], \quad n \geq 1, \end{aligned} \tag{1.4}$$

where $T_n = T_{nmodN}$, converges weakly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$.

THEOREM 1.3. *Let H be a Hilbert space, $F\colon H \to H$ be a mapping such that for some constants $k, \eta > 0$, F is k-Lipschitzian and η-strongly monotone. Let $\{T_i\}_{i=1}^N$ be N nonexpansive self-maps of H such that $C = \bigcap_{i=1}^N F(T_i) \neq \emptyset$. Let $\mu \in (0, 2\eta/k^2)$, let $x_0 \in H$, $\{\lambda_n\}_{n=1}^{\infty} \subset [0, 1)$ and $\{\alpha_n\}_{n=1}^{\infty} \subset (0, 1)$ satisfying the conditions: $\sum_{n=1}^{\infty} \lambda_n < \infty$ and $\alpha \leq \alpha_n \leq \beta$, $n \geq 1$, for some $\alpha, \beta \in (0, 1)$. Then the sequence $\{x_n\}$ defined by (1.4) converges strongly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$ if and only if*

$$\liminf_{n\to\infty} d\left(x_n, \bigcap_{i=1}^N F(T_i)\right) = 0.$$

Xu with Ori [12], and Osilike [9] showed that if X is a Hilbert space, and the sequence $\{x_n\}$ defined by

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)T_n(x_n), \quad x_0 \in C,$$

then $\{x_n\}$ converges weakly to a common fixed point of $\{T_i\}_{i=1}^N$. Chen, Lin, and Song [4] extended the above result to a Banach spaces.

Theorem 1.4. *Let K be a nonempty closed convex subset of a q-uniformly smooth and p-uniformly convex Banach space E that has the Opial property. Let s be any element in $(0,1)$ and let $\{T_i\}_{i=1}^N$ be a finite family of strictly pseudocontractive self-maps of K such that T_i, $1 \le i \le N$ have at least one common fixed point. For any point x_0 in K and any sequence $\{\alpha_n\}_{n=0}^{\infty}$, in $(0,s)$, the sequence*

$$x_n = \alpha_{n-1}x_{n-1} + (1-\alpha_n)T_n x_n,$$

converges weakly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$.

Further, Gu [5] introduced a new composite implicit iteration process as follows:

$$\begin{aligned} x_n &= (1-\alpha_n-\gamma_n)x_{n-1} + \alpha_n T_n(y_n) + \gamma_n u_n, \quad n \ge 1, \\ y_n &= (1-\beta_n-\delta_n)x_n + \beta_n T_n(x_n) + \delta_n v_n, \quad n \ge 1, \end{aligned} \tag{1.5}$$

where $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}, \{\delta_n\}$ are four real sequences in $[0,1]$ satisfying $\alpha_n + \gamma_n \le 1$ and $\beta_n + \delta_n \le 1$ for all $n \ge 1$, $\{u_n\}$ and $\{v_n\}$ are two bounded sequences in C and x_0 is a given point. The following theorem was proved.

Theorem 1.5. *Let X be a real Banach space and C be a nonempty closed convex subset of X. Let $\{T_i\}_{i=1}^N$ be N strictly pseudocontractive mappings of C into C with $S := \bigcap_{i=1}^N F(T_i) \ne \emptyset$. Let $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}, \{\delta_n\}$ be four real sequences in $[0,1]$ satisfying $\alpha_n + \gamma_n \le 1$ and $\beta_n + \delta_n \le 1$ for all $n \ge 1$, $\{u_n\}$ and $\{v_n\}$ be two bounded sequences in C satisfying the following conditions:*

(i) $\sum_{i=1}^{\infty} \alpha_n = \infty$; (ii) $\sum_{i=1}^{\infty} \alpha_n^2 < \infty$; (iii) $\sum_{i=1}^{\infty} \alpha_n\beta_n < \infty$;

(iv) $\sum_{i=1}^{\infty} \alpha_n\delta_n < \infty$; (v) $\sum_{i=1}^{\infty} \gamma_n < \infty$.

Suppose further that and $x_0 \in C$ be a given point and $\{x_n\}$ be the implicit iteration sequence defined by (1.5). Then the following conclusions hold:

(i) $\lim_{n\to\infty} \|x_n - p\|$ *exists for all $p \in S$;*

(ii) $\liminf_{n\to\infty} \|x_n - T_n(x_n)\| = 0$.

Set

$$A_i = I - T_i.$$

Obviously, $S_i := \{x \in X : A_i(x) = 0\} = F(T_i)$ and problem (1.1) is equivalent to the one of finding a common solution of the following operator equations

$$A_i(x) = 0, \quad i = 1, \dots, N,$$

where A_i are Lipschitz continuous and λ_i inverse strongly accretive, i.e. A_i satisfy condition (1.3).

When X is a Hilbert space, another method is considered in [7] and [6] for the case $N = 1$.

In the following section, on the base of [3] we present some new implicit iterative methods of different type which are the Tychonoff regularization method and the regularization inertial proximal point algorithm for solving (1.1) in Banach spaces.

In the sequel, the symbols $\to$ and $\rightharpoonup$ denote the strong and the weak convergence, respectively.

2. Main results

We formulate the following facts needed in the proof of our results.

LEMMA 2.1. [10] *Let $\{a_n\}, \{b_n\}, \{c_n\}$ be sequences of positive numbers satisfying the conditions*

(i) $a_{n+1} \le (1 - b_n)a_n + c_n$, $b_n < 1$,

(ii) $\sum_{n=0}^{\infty} b_n = +\infty$, $\lim_{n\to+\infty} c_n/b_n = 0$.

Then, $\lim_{n\to+\infty} a_n = 0$.

T is said to be *demiclosed at a point* p if whenever $\{x_n\}$ is a sequence in $D(T)$ such that $\{x_n\}$ converges weakly to $x \in D(T)$ and $\{T(x_n)\}$ converges strongly to p, then $T(x) = p$. Furthermore, T is said to be demicompact if whenever $\{x_n\}$ is a bounded sequence in $D(T)$ such that $\{x_n - T(x_n)\}$ converges strongly, then $\{x_n\}$ has a subsequence which converges strongly.

THEOREM 2.1. [8] *Let X be a q-uniformly smooth Banach space which is also uniformly convex. Let K be a nonempty closed convex subset of X and $T\colon K \to K$ a strictly pseudocontractive map. Then $(I - T)$ is demiclosed at zero.*

Consider the operator version of Tychonoff regularization method in the form

$$\sum_{i=1}^{N} \alpha_n^{\mu_i} A_i(x) + \alpha_n x = 0, \tag{2.1}$$
$$\mu_1 = 0 < \mu_i < \mu_{i+1} < 1, \quad i = 1, 2, \ldots, N-1,$$

depending on the positive regularization parameter α_n that tends to zero as $n \to +\infty$. We will prove the following results.

THEOREM 2.2. (i) *For each $\alpha_n > 0$, problem (2.1) has a unique solution x_n.*

(ii) *If one of the following conditions is satisfied:*

(a) X possesses a weak sequential continuous duality mapping j,

(b) there exists a number $i_0 \in \{1, 2, \ldots, N\}$ such that T_{i_0} is demicompact,

then the sequence $\{x_n\}$ possesses a convergent subsequence, and each convergent subsequence of $\{x_n\}$ converges to a solution of (1.1).

(iii) *If the sequence* $\{\alpha_n\}$ *is chosen such that*

$$\lim_{n\to+\infty} \frac{|\alpha_n - \alpha_p|}{\alpha_n} = 0, \quad p > n,$$

then $\lim_{n\to+\infty} x_n = x_*, \quad x_* \in S.$

Proof. (i) Since $\sum_{j=1}^N \alpha_n^{\mu_j} A_j$ is Lipschitz continuous and accretive, then it is m-accretive [1]. Hence, equation (2.1) has a unique solution denoted by x_n for each $\alpha_n > 0$.

From (2.1) it follows

$$\sum_{i=1}^N \alpha_n^{\mu_i} \langle A_i(x_n), j(x_n - y)\rangle + \alpha_n \langle x_n, j(x_n - y)\rangle = 0 \quad \forall y \in S. \tag{2.2}$$

Since $A_i(y) = 0, i = 1, \dots, N$, then

$$\sum_{i=1}^N \alpha_n^{\mu_i} A_i(y) = 0.$$

The last equality, (2.2) and the accretive property of A_i give $\langle x_n, j(x_n - y)\rangle \le 0$, i.e.

$$\langle x_n - y, j(x_n - y)\rangle \le \langle -y, j(x_n - y)\rangle \quad \forall y \in S. \tag{2.3}$$

Consequently,

$$\|x_n - y\| \le \|y\| \quad \text{and} \quad \|x_n\| \le 2\|y\|, \quad y \in S. \tag{2.4}$$

Hence, $\{x_n\}$ is bounded. Let $x_{n_k} \rightharpoonup \tilde{x} \in X$, as $k \to +\infty$. First, we prove that $\tilde{x} \in S_1$. Indeed, by virtue of (2.1), (2.4) and the Lipschitz continuity of A_i with $A_i(y) = 0$, we can write

$$\begin{aligned}\|A_1(x_{n_k})\| &\le \sum_{i=2}^N \alpha_{n_k}^{\mu_i} \|A_i(x_{n_k})\| + \alpha_{n_k}\|x_{n_k}\| \\ &\le L\|y\| \sum_{i=2}^N \alpha_{n_k}^{\mu_i} + 2\alpha_{n_k}\|y\|, \quad L = \max_{1\le i\le N}\{1/\lambda_i\}.\end{aligned}$$

Since A_1 is demiclosed at zero by Theorem 2.1, then $A_1(\tilde{x}) = 0$, i.e., $\tilde{x} = T_1(\tilde{x})$. It means $\tilde{x} \in F(T_1)$.

Now, we shall prove that $\tilde{x} \in F(T_i), i = 2, \dots, N$. Again, for any $y \in S$ from (1.3), (2.1) and the accretive property of A_i imply that

$$\begin{aligned}\|A_2(x_{n_k})\|^2 &\le \langle A_2(x_{n_k}), j(x_{n_k} - y)\rangle/\lambda_2 \\ &\le \sum_{i=3}^N \alpha_{n_k}^{\mu_i - \mu_2} \langle A_i(y) - A_i(x_{n_k}), j(x_{n_k} - y)\rangle/\lambda_2 \\ &\quad + \alpha_{n_k}^{1-\mu_2}\langle -y, j(x_{n_k} - y)\rangle/\lambda_2,\end{aligned}$$

i.e. $\|A_2(x_{n_k})\|^2 \le \alpha_{n_k}^{1-\mu_2}\|y\|^2/\lambda_2$. Therefore,

$$\lim_{k\to\infty} \|A_2(x_{n_k})\|^2 = 0.$$

Again, by virtue of the demiclosed property of A_2, we have $A_2(\tilde{x}) = 0$, i.e., $\tilde{x} \in F(T_2)$.

Set $\tilde{S}_m = \bigcap_{l=1}^m F(T_l)$. Then, $\tilde{S}_m$ is also closed convex, and $\tilde{S}_m \neq \emptyset$. Now, suppose that we have proved $\tilde{x} \in \tilde{S}_m$, and need to show that $\tilde{x}$ belongs to S_{m+1}. By virtue of (2.1), for $y \in S$, we can write

$$\begin{aligned}\|A_{m+1}(x_{n_k})\|^2 &\leq \langle A_{m+1}(x_{n_k}, j(x_{n_k} - y))\rangle/\lambda_{m+1}\\ &\leq \sum_{i=m+2}^{N} \alpha_{n_k}^{\mu_i - \mu_{m+1}} \langle A_i(y) - A_i(x_{n_k}), j(x_{n_k} - y)\rangle/\lambda_{m+1}\\ &\quad + \alpha_{n_k}^{1-\mu_{m+1}} \langle -y, j(x_{n_k} - y)\rangle/\lambda_{m+1},\end{aligned}$$

i.e. $\|A_{m+1}(x_{n_k})\|^2 \leq \alpha_{n_k}^{1-\mu_m} \|y\|^2/\lambda_{m+1}$. Hence,

$$\lim_{k\to\infty} \|A_{m+1}(x_{n_k})\|^2 = 0.$$

Again, since A_{m+1} is demiclosed at zero, then $\tilde{x} \in S_{m+1}$. It means that $\tilde{x} \in S$.

From the weak sequential continuous property of the duality mapping j and (2.3) with $y = \tilde{x}$ or the demicompact property of T_{i_0} it follows

$$\lim_{k\to+\infty} x_{n_k} = \tilde{x} \in S.$$

(iii) Let x_p be the solution of (2.1) when α_n is replaced by α_p. Then,

$$\sum_{i=1}^{N} (\alpha_n^{\mu_i} - \alpha_p^{\mu_i}) \langle A_i(x_n), j(x_n - x_p)\rangle + \alpha_n \langle x_n, j(x_n - x_p)\rangle + \alpha_p \langle x_p, j(x_p - x_n)\rangle \leq 0.$$

Hence,

$$\|x_n - x_p\| \leq \frac{|\alpha_n - \alpha_p|}{\alpha_n} 2\|y\| + \frac{L\|y\|}{\alpha_n} \sum_{i=2}^{N} |\alpha_n^{\mu_i} - \alpha_p^{\mu_i}|.$$

Using the Lagrange's mean-value theorem for the function $\alpha(t) = t^{-\mu}$, $0 < \mu < 1$, $t \in [1, +\infty)$ on the interval $[a, b]$ with $a = \alpha_n, b = \alpha_p$ or $a = \alpha_p, b = \alpha_n$, we have the estimation

$$\|x_n - x_p\| \leq M \frac{|\alpha_n - \alpha_p|}{\alpha_n}$$

with $M = 2\|y\| + L\|y\|(N - 1)$. Clearly, if

$$\lim_{n\to+\infty} \frac{|\alpha_n - \alpha_p|}{\alpha_n} = 0, \quad p > n,$$

then $\{x_n\}$ is a Cauchy sequence in the Banach space X. Therefore, $\lim_{n\to+\infty} x_n = x_* \in S$. The theorem is proved. ■

THEOREM 2.3. *Assume that parameters $\tilde{c}_n$, γ_n and α_n are chosen such that*

(i) $0 < c_0 < \tilde{c}_n < C_0$, $0 \leq \gamma_n < \gamma_0$, $\alpha_n \searrow 0$,

(ii) $\sum_{n=1}^{\infty} b_n = +\infty$, $b_n = \alpha_n \tilde{c}_n/(1 + \alpha_n \tilde{c}_n)$, $\sum_{n=1}^{\infty} \gamma_n b_n^{-1} \|z_n - z_{n-1}\| < +\infty$,

(iii) $\lim_{n\to\infty} \frac{\alpha_n - \alpha_{n+1}}{\alpha_n b_n} = 0$.

Then, the sequence $\{z_n\}$ *defined by*

$$\tilde{c}_n\left(\sum_{i=1}^{N} \alpha_n^{\mu_i} A_i(z_{n+1}) + \alpha_n z_{n+1}\right) + z_{n+1} - z_n = \gamma_n(z_n - z_{n-1}), \qquad z_0, z_1 \in X, \tag{2.5}$$

converges to an element in S.

Proof. We rewrite equations (2.5) and (2.1) in their equivalent forms

$$d_n \sum_{i=1}^{N} \alpha_n^{\mu_i} A_i(z_{n+1}) + z_{n+1} = \beta_n z_n + \beta_n \gamma_n (z_n - z_{n-1}),$$
$$d_n \sum_{i=1}^{N} \alpha_n^{\mu_i} A_i(x_n) + x_n = \beta_n x_n,$$
$$d_n = \beta_n \tilde{c}_n, \beta_n = 1/(1 + \alpha_n \tilde{c}_n).$$

After subtracting the second equality from the first one and multiplying by $j(z_{n+1} - x_n)$ we get

$$\begin{aligned} d_n \langle \sum_{i=1}^{N} \alpha_n^{\mu_i} (A_i(z_{n+1}) - A_i(x_n)), j(z_{n+1} - x_n)\rangle + \langle z_{n+1} - x_n, j(z_{n+1} - x_n)\rangle \\ = \beta_n \langle z_n - x_n, j(z_{n+1} - x_n)\rangle + \beta_n\gamma_n \langle z_n - z_{n-1}, j(z_{n+1} - x_n)\rangle. \end{aligned}$$

Again, by virtue of the property of A_i and j it is not difficult to verify the following inequality

$$\|z_{n+1} - x_n\| \le \beta_n \|z_n - x_n\| + \beta_n\gamma_n \|z_n - z_{n-1}\|.$$

Consequently,

$$\begin{aligned} \|z_{n+1} - x_{n+1}\| &\le \|z_{n+1} - x_n\| + \|x_{n+1} - x_n\| \\ &\le \beta_n \|z_n - x_n\| + \beta_n\gamma_n \|z_n - z_{n-1}\| + M\frac{\alpha_n - \alpha_{n+1}}{\alpha_n} \\ &\le (1 - b_n)\|z_n - x_n\| + c_n, \end{aligned}$$

where $c_n = \beta_n\gamma_n\|z_n - z_{n-1}\| + M(\alpha_n - \alpha_{n+1})/\alpha_n$. Since the series in (ii) is convergent, $\gamma_n b_n^{-1}\|z_n - z_{n-1}\| \to 0$, as $n \to +\infty$. Lemma 2.1 guarantees that $\|z_{n+1} - x_{n+1}\| \to 0$ as $n \to +\infty$. On the other hand, from

$$\lim_{n\to+\infty} \frac{\alpha_n - \alpha_{n+1}}{\alpha_n b_n} = 0,$$

it follows that $\lim_{n\to+\infty} \dfrac{\alpha_n - \alpha_{n+1}}{\alpha_n} = 0$. Therefore, for any $p > n$ we have

$$\begin{aligned} 0 \le \lim_{n\to+\infty} \frac{\alpha_n - \alpha_p}{\alpha_n} &= \lim_{n\to+\infty} \left[\frac{\alpha_n - \alpha_{n+1}}{\alpha_n} + \frac{\alpha_{n+1} - \alpha_{n+2}}{\alpha_n} + \cdots + \frac{\alpha_{p-1} - \alpha_p}{\alpha_n}\right] \\ &\le \lim_{n\to+\infty} \left[\frac{\alpha_n - \alpha_{n+1}}{\alpha_n} + \frac{\alpha_{n+1} - \alpha_{n+2}}{\alpha_{n+1}} + \cdots + \frac{\alpha_{p-1} - \alpha_p}{\alpha_{p-1}}\right] \\ &= 0 + \cdots + 0 = 0. \end{aligned}$$

Theorem 2.2 permits us to conclude that the sequence $\{x_n\}$ converges to an element in S. Hence, $\{z_n\}$ also converges to an element in S. The theorem is proved. ∎

REMARK The sequences $\{\alpha_n\}$ and $\{\gamma_n\}$ defined by $\alpha_n = \alpha_0(1+n)^{-\alpha}$ and

$$\gamma_n = b_0(1+n)^{-\gamma}\frac{1}{1+\|z_n - z_{n-1}\|},$$

where α_0, b_0 are some positive constants and $0 < \alpha < \frac{1}{2}$, $\gamma > \alpha + 1$, satisfy all conditions in the theorem.

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